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Resistance Calculation for an infinite Simple Cubic Lattice Application of Green's Function

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It is shown that the resistance between the origin and any lattice point (l, m, n) in an infinite perfect Simple Cubic (SC) lattice is expressible rationally in terms of the known value of $G_0(0, 0, 0)$. The resistance between arbitrary sites in an infinite SC lattice is also studied and calculated when one of the resistors is removed from the perfect lattice. The asymptotic behavior of the resistance for both the infinite perfect and perturbed SC lattice is also investigated. Finally, experimental results are obtained for a finite SC network consisting of $8 \times 8 \times 8$ identical resistors, and a comparison with those obtained theoretically is presented.

KEY WORDS: Lattice Green's Function; resistors; simple cubic lattice.

1. INTRODUCTION

The calculation of the resistance between two arbitrary grid points of infinite networks of resistors is a new-old subject (Van der Pol and Bremmer, 1955; Doyle and Snell, 1984; Venezian, 1994; Atkinson and Van Steenwijk, 1999; Aitchison, 1964; Bartis, 1967; Monwhea, 2000).

Recently, Cserti (2000) and Cserti *et al.* (2002) studied the problem where they introduced a method based on the Lattice Green's Function (LGF) which is an alternative approach to using the superposition of current distributions presented by Venezian (1994) and (Atkinson and Van Steenwijk, 1999).

The LGF for cubic lattices has been investigated by many authors (Morita and Horiguchi, 1975; Joyce, 1971; Sakaji *et al.*, 2002; Hijjawi and Khalifeh, 2002; Sakaji *et al.*, 2002; Hijjawi and Khalifeh, 2002; Morita and Horigucih, 1971; Inoue, 1975; Mano, 1975; Katsura and Horiguchi, 1971; Glasser, 1972), and the so-called recurrence formulae which are often used to calculate the LGF of the SC at different sites are presented (Glasser, 1972; Horiguchi, 1971).

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The values of the LGF for the SC lattice have been recently exactly evaluated (Glasser and Boersma, 2000), where these values are expressed in terms of the known value of the LGF at the origin.

In this paper; we calculate the resistance between two arbitrary points in a perfect and perturbed (i.e. a bond is removed) infinite SC lattice using Cserti's method (Cserti, 2000; Cserti *et al.*, 2002). The resistance between the origin and a lattice site (l, m, n) in a constructed finite perfect SC mesh ($8 \times 8 \times 8$ resistors) is measured. Also, the resistance between the origin and a lattice site (l, m, n) in the same constructed mesh, when one of the resistors is removed (i.e. perturbed) is measured. Finally, a comparison is carried out between the measured resistances and those calculated by Cserti's method (Cserti, 2000; Cserti *et al.*, 2002).

The LGF presented here is related to the LGF of the Tight-Binding Hamiltonian (TBH) (Economou and Green's Function in Quantum Physics, 1983).

2. THEORETICAL RESULTS

2.1. Perfect SC Lattice

In this section we express the resistance in a perfect infinite SC network of identical resistors between the origin and any lattice site (l, m, n) rationally as: (Cserti, 2000; Glasser and Boersma, 2000)

$$\frac{R_0(l, m, n)}{R} = \rho_1 g_0 + \frac{\rho_2}{\pi^2 g_0} + \rho_3 \tag{1}$$

where $g_0 = G_0(0, 0, 0)$ is the LGF at the origin and ρ_1, ρ_2, ρ_3 are related to r_1, r_2, r_3 or Duffin and Shelly's (Glasser and Boersma, 2000; Duffin and Shelly, 1958) parameters $\lambda_1, \lambda_2, \lambda_3$ as

$$\rho_1 = 1 - r_1 = 1 - \lambda_1 - \frac{15}{12}\lambda_2; \tag{2}$$

$$\rho_2 = -r_2 = \frac{1}{2}\lambda_2; \tag{3}$$

$$\rho_3 = -r_3 = \frac{1}{3}\lambda_3. \tag{4}$$

Various values of r_1, r_2, r_3 are shown in Glasser and Boersma (Glasser and Boersma, 2000) [Table I] for (l, m, n) ranging from $(0, 0, 0) - (5, 5, 5)$. To obtain other values of r_1, r_2, r_3 one has to use the relation (Horiguchi, 1971)

$$\begin{aligned} &G_0(l+1, m, n) + G_0(l-1, m, n) + G_0(l, m, +1, n) + G_0(l, m-1, n) + \\ &G_0(l, m, n+1) + G_0(l, m, n-1) = -2\delta_{l0}\delta_{m0}\delta_{n0} + 2EG_0(l, m, n). \end{aligned} \tag{5}$$

where $E = 3$, is the energy.

Table I. Values of the resistance in a perfect infinite SC lattice for arbitrary sites

Site l, m, n	ρ_1	ρ_2	ρ_3	$\frac{R_0(l,m,n)}{R} = \rho_1 g_0 + \frac{\rho_2}{\pi^2 g_0} + \rho_3$
000	0	0	0	0
100	0	0	1/3	0.333333
110	7/12	1/2	0	0.395079
111	9/8	−3/4	0	0.418305
200	−7/3	−2	2	0.419683
210	5/8	9/4	−1/3	0.433598
211	5/3	−2	0	0.441531
220	−37/36	29/6	0	0.449351
221	31/16	−21/8	0	0.453144
222	3/8	27/20	0	0.460159
300	−33/2	−21	13	0.450371
310	115/36	85/6	−4	0.454415
311	15/4	−21/2	2/3	0.457396
320	−271/48	119/8	1/3	0.461311
321	161/36	−269/30	0	0.463146
322	−19/16	213/40	0	0.467174
330	−47/3	1046/25	0	0.468033
331	38/3	−148/5	0	0.469121
332	−26/9	1012/105	0	0.471757
333	51/16	−1587/280	0	0.475023
400	−985/9	−542/3	92	0.464885
410	531/16	879/8	−115/3	0.466418
411	11/2	−357/5	12	0.467723
420	−2111/72	13903/300	6	0.469777
421	245/16	−1251/40	−1	0.470731
422	−32/3	1024/35	0	0.473076
430	−2593/48	28049/200	−1/3	0.473666
431	1541/36	−110851/1050	0	0.474321
432	−493/32	4617/112	0	0.476027
433	667/72	−8809/420	0	0.478288
440	−5989/36	620161/1470	0	0.477378
441	4197/32	−919353/2800	0	0.477814
442	−2927/48	31231/200	0	0.479027
443	571/32	−119271/2800	0	0.480700
444	−69/8	186003/7700	0	0.482570
500	−9275/12	−3005/2	2077/3	0.473263
510	11653/36	138331/150	−348	0.473986
511	−271/4	−5751/10	150	0.474646
520	−2881/16	15123/200	229/3	0.475807
521	949/12	−27059/350	−24	0.476341
522	−501/8	4209/28	2	0.477766
530	−3571/18	1993883/3675	−8	0.478166
531	1337/8	−297981/700	4/3	0.478565
532	−2519/36	187777/1050	0	0.479693
533	2281/48	−164399/1400	0	0.481253
540	−18439/32	28493109/19600	1/3	0.480653

Table I. Continued

Site l, m, n	ρ_1	ρ_2	ρ_3	$\frac{R_0(l,m,n)}{R} = \rho_1 g_0 + \frac{\rho_2}{\pi^2 g_0} + \rho_3$
541	1393/3	−286274/245	0	0.480920
542	−7745/32	1715589/2800	0	0.481798
543	5693/72	−4550057/23100	0	0.483012
544	−1123/32	560001/6160	0	0.484441
550	−196937/108	101441689/22050	0	0.483050
551	12031/8	−18569853/4900	0	0.483146
552	−1681/2	5718309/2695	0	0.483878
553	5175/16	−2504541/3080	0	0.484777
554	−24251/312	−1527851/7700	0	0.485921
555	9459/208	−12099711/107800	0	0.487123
600	−34937/6	−313079/25	5454	0.478749
610	71939/24	160009/20	−9355/3	0.479137
633	18552/72	−747654/1155	0	0.483209
644	−388051/1872	23950043/46200	0	0.486209
655	13157/78	−5698667/13475	0	0.488325
700	−553847/12	5281913/50	44505	0.482685

In some cases one may need to use the recurrence formulae (i.e. Equation (5)) two or three times to calculate different values of r_1, r_2, r_3 for (l, m, n) beyond $(5, 5, 5)$. Various values of ρ_1, ρ_2, ρ_3 are shown in Table I.

The value of the LGF at the origin (i.e. $G_0(0, 0, 0)$) was first evaluated by Watson in his famous paper (Watson, 1939), where he found that

$$G_0(0,0,0) = \left(\frac{2}{\pi}\right)^2 (18 + 12\sqrt{2} - 10\sqrt{3} - 7\sqrt{6})[K(k_0)]^2 = 0.505462.$$

with $k_0 = (2 - \sqrt{3})(\sqrt{3} - \sqrt{2})$ and $K(k) = \int_0^{\frac{\pi}{2}} d\theta \frac{1}{\sqrt{1 - k^2 Sin^2\theta}}$ is the complete elliptic integral of the first kind.

A similar result was obtained by Glasser and Zucker (1977) in terms of gamma function.

The asymptotic behavior (i.e. as l, or m, or $n \to \infty$) of the resistance in a perfect infinite SC is (see Appendix A)

$$\frac{R_0(l,m,n)}{R} \to g_0. \tag{6}$$

2.2. Perturbed SC Lattice

In this section, we calculate the resistance between any two lattice sites in an infinite SC network of identical resistors, when one of the resistors (i.e. bonds)

Table II. Calculated and measured values of the resistance between the sites $i = (0, 0, 0)$ and $j = (j_x, j_y, j_z)$, for a perturbed simple cubic lattice (i.e. the bond between $i_0 = (0, 0, 0)$ and $j_0 = (1, 0, 0)$ is broken)

The Site $j = (j_x, j_y, j_z)$	$\frac{R(i,j)}{R}$ Theoretically	$\frac{R(i,j)}{R}$ Experimentally	The Site $j = (j_x, j_y, j_z)$	$\frac{R(i,j)}{R}$ Theoretically	$\frac{R(i,j)}{R}$ Experimentally
(0,0,0)	0	0	(−1,0,0)	0.356208	0.3559
(1,0,0)	0.5	0.5009	(−2,0,0)	0.454031	0.4565
(2,0,0)	0.485733	0.4904	(−3,0,0)	0.4526508	0.5003
(3,0,0)	0.500062	0.5151	(−4,0,0)	0.467337	0.5699
(4,0,0)	0.510257	0.5806	(0,−1,0)	0.360993	0.3606
(0,1,0)	0.360993	0.3615	(0,−2,0)	0.457943	0.4611
(0,2,0)	0.457943	0.4612	(0,−3,0)	0.491033	0.5040
(0,3,0)	0.491033	0.5041	(0,−4,0)	0.506167	0.5735
(0,4,0)	0.506167	0.5735	(0,0,−1)	0.360993	0.3613
(0,0,1)	0.360993	0.3611	(0,0,−2)	0.457943	0.4615
(0,0,2)	0.457943	0.4613	(0,0,−3)	0.491033	0.5043
(0,0,3)	0.491033	0.5042	(0,0,−4)	0.506167	0.5736
(0,0,4)	0.506167	0.5737	(−1,−1,−1)	0.454367	0.4560
(1,1,1)	0.4659804	0.4203	(−2,−2,−2)	0.50009	0.5170
(2,2,2)	0.503597	0.4780	(−3,−3,−3)	0.5158855	0.5854
(3,3,3)	0.517510166	0.5458	(−4,−4,−4)	0.5237707	0.8974
(4,4,4)	0.524705	0.8579			

between the sites $i_0 = (i_{0x}, i_{0y}, i_{0z})$ and $j_0 = (j_{0x}, j_{0y}, j_{0z})$ is removed (Cserti *et al.*, 2002), where

$$R(i, j) = R_0(i, j) + \frac{[R_0(i, j_0) + R_0(j, i_0) - R_0(i, i_0) - R_0(j, j_0)]^2}{4[R - R_0(i_0, j_0)]} \tag{7}$$

As an example; let us assume that the bond between $i_0 = (0, 0, 0)$ and $j_0 = (1, 0, 0)$ is removed. So, we calculate the resistance between any two sites. Our results are arranged in Table II, and for example:

The resistance between the sites $i = (0, 0, 0)$ and $j = (1, 0, 0)$ is

$$R(1, 0, 0) = \frac{R}{2}. \tag{8}$$

i.e. the resistance between the two ends of the removed bond is $\frac{R}{2}$, which is a predictable result (Cserti *et al.*, 2002).

Now, if the removed bond is shifted and set between the sites $i_0 = (1, 0, 0)$ and $j_0 = (2, 0, 0)$**,** then one can find the resistance between any two sites i.e. $i = (i_x, i_y, i_z)$ and $j = (j_x, j_y, j_z)$). Using Equation (7) again one obtains the results arranged in Table III.

Table III. Calculated and measured values of the resistance between the sites $i = (0, 0, 0)$ and $j = (j_x, j_y, j_z)$, for a perturbed SC lattice (i.e. the bond between $i_0 = (1, 0, 0)$ and $j_0 = (2, 0, 0)$ is broken)

The Site $j = (j_x, j_y, j_z)$	$\frac{R(i,j)}{R}$ Theoretically	$\frac{R(i,j)}{R}$ Experimentally	The Site $j = (j_x, j_y, j_z)$	$\frac{R(i,j)}{R}$ Theoretically	$\frac{R(i,j)}{R}$ Experimentally
(0,0,0)	0	0	(−1,0,0)	0.334495	0.3345
(1,0,0)	0.356208	0.3552	(−2,0,0)	0.421618	0.4247
(2,0,0)	0.485733	0.4903	(−3,0,0)	0.452650	0.4656
(3,0,0)	0.461555	0.4757	(−4,0,0)	0.467337	0.5342
(4,0,0)	0.470021	0.5389	(0,−1,0)	0.334191	0.3338
(0,1,0)	0.334191	0.3346	(0,−2,0)	0.421552	0.4247
(0,2,0)	0.421552	0.4247	(0,−3,0)	0.452738	0.4656
(0,3,0)	0.452738	0.4657	(0,−4,0)	0.467467	0.5348
(0,4,0)	0.467467	0.5347	(−1,−1,−1)	0.420168	0.4185
(1,1,1)	0.419799	0.4218	(−2,−2,−2)	0.462590	0.4795
(2,2,2)	0.460461	0.4812	(−3,−3,−3)	0.477628	0.5479
(3,3,3)	0.477922	0.5494	(−4,−4,−4)	0.485253	0.8602
(4,4,4)	0.485476	0.8616			

For large separation between the sites i *and* j the resistance in an infinite perturbed SC lattice becomes (see Appendix B).

$$\frac{R(i, j)}{R} \rightarrow \frac{R_0(i, j)}{R} = g_0. \tag{9}$$

That is, the resistance between the sites i *and* j in an infinite perturbed SC lattice goes to a finite value.

3. EXPERIMENTAL RESULTS

To study the resistance of the SC lattice experimentally we constructed a three-dimensional SC finite network consisting of $(8 \times 8 \times 8)$ identical resistors, each has a value of (1 kΩ) and tolerance (1%).

3.1. Perfect Case

Using the constructed perfect mesh we measured the resistance between the origin and the site (l, m, n) along the directions [100], [010], [001], and [111]. Our results are arranged in Table IV.

3.2. Perturbed Case

To measure the resistance for the perturbed case we removed the bond between $i_0 = (0, 0, 0)$ and $j_0 = (1, 0, 0)$ in the constructed mesh, then we measured the resistance between the site $i = (0, 0, 0)$ and the site $j = (j_x, j_y, j_z)$

Table IV. Calculated and measured values of the resistance between the origin and an arbitrary site in a perfect SC lattice

The Site (l,m,n)	$\frac{R_0(l,m,n)}{R}$ Theoretically	$\frac{R_0(l,m,n)}{R}$ Experimentally	The Site (l,m,n)	$\frac{R_0(l,m,n)}{R}$ Theoretically	$\frac{R_0(l,m,n)}{R}$ ly Experimentally
(0,0,0)	0	0	(−1,0,0)	0.3333	0.3333
(1,0,0)	0.3333	0.3331	(−2,0,0)	0.419683	0.4230
(2,0,0)	0.419683	0.4227	(−3,0,0)	0.450371	0.4635
(3,0,0)	0.450371	0.4633	(−4,0,0)	0.464885	0.5321
(4,0,0)	0.464885	0.5323	(0,−1,0)	0.3333	0.3337
(0,1,0)	0.3333	0.3331	(0,−2,0)	0.419683	0.4228
(0,2,0)	0.419683	0.4228	(0,−3,0)	0.450371	0.4634
(0,3,0)	0.450371	0.4623	(0,−4,0)	0.464885	0.5322
(0,4,0)	0.464885	0.5321	(0,0,−1)	0.3333	0.3335
(0,0,1)	0.3333	0.3334	(0,0,−2)	0.419683	0.4231
(0,0,2)	0.419683	0.4230	(0,0,−3)	0.450371	0.4635
(0,0,3)	0.450371	0.4634	(0,0,−4)	0.464885	0.5324
(0,0,4)	0.464885	0.5325	(−1,−1,−1)	0.418305	0.4204
(1,1,1)	0.418305	0.4203	(−2,−2,−2)	0.460159	0.4772
(2,2,2)	0.460159	0.4774	(−3,−3,−3)	0.475023	0.5464
(3,3,3)	0.475023	0.5461	(−4,−4,−4)	0.482570	0.8583
(4,4,4)	0.482570	0.8581			

along the directions [100], [010], [001], and [111]**.** Our results are arranged in Table II.

Now, the removed bond is shifted, $i_0 = (1, 0, 0)$ and $j_0 = (2, 0, 0)$, then we again measured the resistance between the site $i = (0, 0, 0)$ and the site $j = (j_x, j_y, j_z)$ along the directions [100], [010], [001], and [111]**.** Our results are arranged in Table III.

4. RESULTS AND DISCUSSION

From the Figures shown the resistance in an infinite SC lattice is symmetric under the transformation $(l, m) \to (-l, -m)$ due to the inversion symmetry of the lattice. However, the resistance in the perturbed infinite SC lattice is not symmetric due to the removed bond.

Also, one can see that the resistance in the perturbed infinite SC lattice is always larger than that in a perfect lattice and this is due to the positive second term in Equation (7). But as the separation between the sites increases the perturbed resistance goes to that of a perfect lattice more rapidly. This means that the effect of the perturbation decreases.

Figure 1 shows the resistance against the site (l, m, n) along the [100] direction for both a perfect infinite and perturbed SC (i.e. the bond between $i_0 = (0, 0, 0)$ and $j_0 = (1, 0, 0)$ is broken). It is seen from the figure that the resistance is symmetric

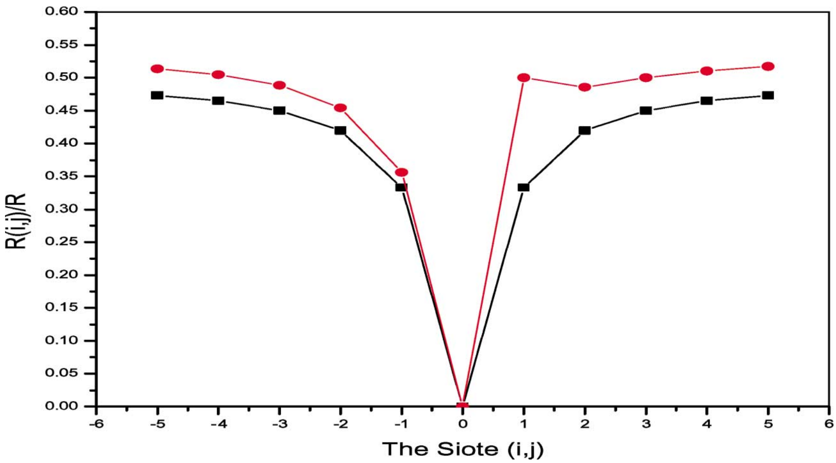

Fig. 1. The resistance on the perfect (squares) and the perturbed (circles) SC between $i = (0, 0, 0)$ and $j = (j_x, 0, 0)$ along the [100] direction as a function of j_x. The ends of the removed bond are $i_0 = (0, 0, 0)$ and $j_0 = (1, 0, 0)$.

(i.e. $R_0(l, 0, 0) = R_0(-l, 0, 0)$) for the perfect case due to the inversion symmetry of the lattice while for the perturbed case the symmetry is broken so, the resistance is not symmetric. As (l, m, n) goes away from the origin the resistance approaches its finite value for both cases.

Figure 2 shows the resistance against the site (l, m, n) along the [100] direction for both a perfect infinite and perturbed SC (i.e. the bond between $i_0 = (1, 0, 0)$ and $j_0 = (2, 0, 0)$ is removed). It is seen from the figure that the resistance is symmetric (i.e. $R_0(l, 0, 0) = R_0(-l, 0, 0)$) for the perfect case due to inversion symmetry of the lattice while for the perturbed case the symmetry is broken, hence the resistance is not symmetric. As (l, m, n) goes away from the origin the resistance approaches a finite value for both cases.

Figure 3 shows the measured and calculated resistances of the perfect SC lattice against the site (l, m, n) along the [100] direction. It is seen from the figure that the measured resistance is symmetric within the experimental error (i.e. $R_0(l, 0, 0) = R_0(-l, 0, 0)$) due to inversion symmetry of the mesh.

Figure 4 shows the measured and calculated resistance values of the perturbed (i.e. the bond between $i_0 = (0, 0, 0)$ and $j_0 = (1, 0, 0)$ is broken) SC lattice against the site (l, m, n) along the [100] direction. It is seen from the figure that the measured resistance is not symmetric (i.e. $R_0(l, 0, 0) \neq R_0(-l, 0, 0)$) due to the removed bond.

Figure 5 shows the measured and calculated resistance of the perturbed (i.e. the bond between $i_0 = (1, 0, 0)$ and $j_0 = (2, 0, 0)$ is broken) SC lattice against the site (l, m, n) along the [100] direction. It is seen from the figure that the measured

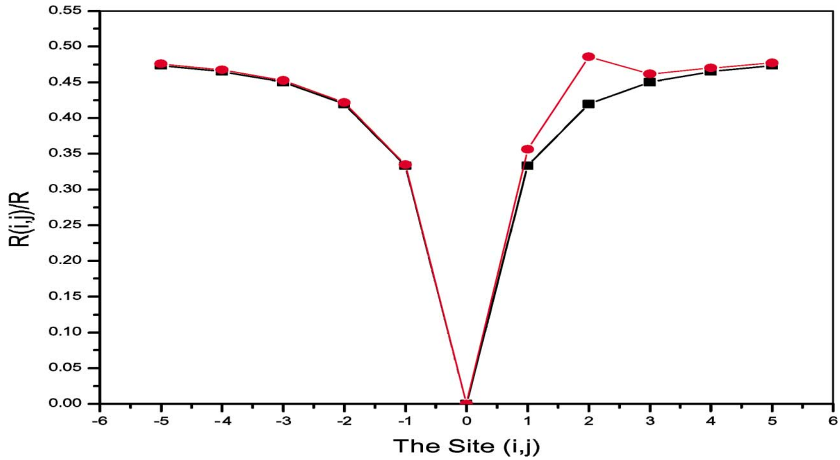

Fig. 2. The resistance on the perfect (squares) and the perturbed (circles) SC between $i = (0, 0, 0)$ and $j = (j_x, 0, 0)$ along the [100] direction as a function of j_x. The ends of the removed bond are $i_0 = (1, 0, 0)$ and $i_0 = (2, 0, 0)$.

resistance is not symmetric (i.e. $R_0(l, 0, 0) \neq R_0(-l, 0, 0)$) due to the removed bond.

From Figs. (1–5) the $(8 \times 8 \times 8)$ constructed finite SC mesh gives the measured bulk resistance nearly same as those calculated. This also shows that one can

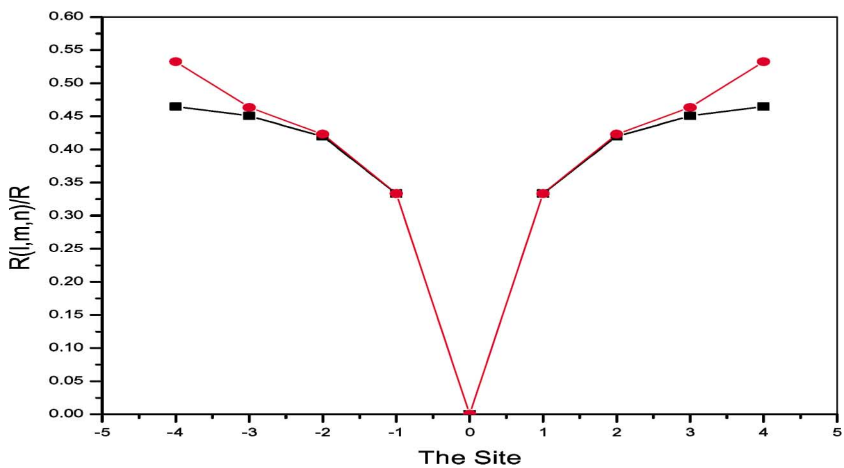

Fig. 3. The resistance between $i = (0, 0, 0)$ and $j = (j_x, 0, 0)$ of the perfect SC lattice as a function of j_x; calculated (squares) and measured (circles) along the [100] direction.

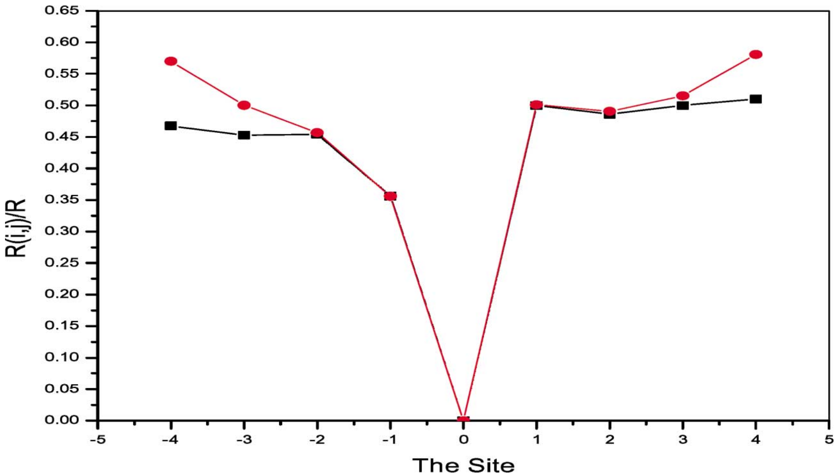

Fig. 4. The resistance between $i = (0, 0, 0)$ and $j = (j_x, 0, 0)$ of the perturbed SC as a function of j_x; calculated (squares) and measured (circles) along the [100] direction. The ends of the removed bond are $i_0 = (0, 0, 0)$ and $j_0 = (1, 0, 0)$.

study the bulk properties of a crystal consisting of $(8 \times 8 \times 8)$ atoms accurately. In addition, as we approach the surface of the SC mesh the measured resistance exceeds the calculated due to surface effect.

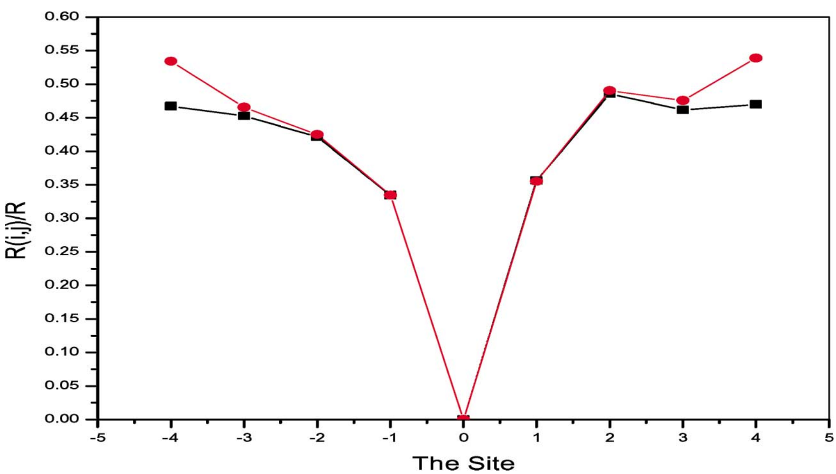

Fig. 5. The resistance between $i = (0, 0, 0)$ and $j = (j_x, 0, 0)$ of the perturbed SC as a function of j_x; calculated (squares) and measured (circles) along the [100] direction. The ends of the removed bond are $i_0 = (1, 0, 0)$ and $j_0 = (2, 0, 0)$.

APPENDIX A

Asymptotic Form of the Resistance for an Infinite Perfect SC Lattice

The resistance between the origin and any lattice site (l, m, n) in an infinite perfect SC lattice is given as (Cserti, 2000):

$$\frac{R_0(l,m,n)}{R} = [G_0(0,0,0) - G_0(l,m,n)] \tag{A1}$$

Now, the LGF for a perfect SC lattice is given as [Economou, 1983]

$$G_0(l,m,n) = \left(\frac{1}{\pi^3}\right)\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \frac{\cos lx \cos my \cos nz}{E - \cos x - \cos y - \cos z} dxdydz \tag{A2}$$

Taking the limit of Equation (A2) as $l \to \infty$, then we may write

$$\operatorname*{Lim}_{l\to\infty} G_o(l,m,n) = \left(\frac{1}{\pi^3}\right)\operatorname*{Lim}_{l\to\infty}\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \frac{\cos lx \cos my \cos nz}{E - (\cos x + \cos y + \cos z)} dxdydz \tag{A3}$$

$$= \left(\frac{1}{\pi^3}\right)\int_0^{\pi}\int_0^{\pi}[\operatorname*{Lim}_{l\to\infty}\int_0^{\pi} \frac{\cos lx}{E - (\cos x + \cos y + \cos z)} dx] \cos my \cos nzdydz \tag{A4}$$

Now, take I to be

$$\begin{aligned} I &= \operatorname*{Lim}_{l\to\infty}\int_0^{\pi} \frac{\cos lx}{E - (cox + \cos y + \cos z)} dx; \\ &= \operatorname*{Lim}_{l\to\infty}\int_0^{\pi} \phi(x) \cos lxdx. \end{aligned} \tag{A5}$$

In the theory of Fourier series, we have the so-called Riemann's lemma i.e.:

$$\operatorname*{Lim}_{p\to\infty}\int_a^b \phi(x)\cos pxdx \to 0. \tag{A6}$$

From Equation (A6), we conclude that$I = 0$. Thus, Equation (A4) becomes

$$\operatorname*{Lim}_{l\to\infty} G_o(l,m,n) \to 0. \tag{A7}$$

The same thing can be done for $m \to \infty$ and for $n \to \infty$. Thus, we conclude that the LGF for a perfect SC lattice goes to zero as any of l, or m, or n goes to infinity. Finally, Equation (A1) becomes

$$\frac{R_0(l,m,n)}{R} \to G_0(0,0,0). \tag{A8}$$

So the resistance in a perfect SC lattice goes to a finite value for large separation between the origin and the site (l, m, n).

APPENDIX B

Asymptotic Form of the Resistance for an Infinite Perturbed SC Lattice

The resistance between the site $i = (i_x, i_y, i_z)$ and the site $j = (j_x, j_y, j_z)$ in an infinite perturbed SC lattice is given as:

$$R(i, j) = R_0(i, j) + \frac{[R_0(i, j_0) + R_0(j, i_0) - R_0(i, i_0) - R_0(j, j_0)]^2}{4[R - R_0(i_0, j_0)]}. \quad \text{(B1)}$$

where the resistor between the sites $i_0 = (i_{0x}, i_{0y}, i_{0z})$ and $j_0 = (j_{0x}, j_{0y}, j_{0z})$ is broken.

Substituting Equation (A1) into the nominator of Equation (B1), we get

$$R(i, j) = R_0(i, j) + \frac{R[-G_0(i, j_0) - G_0(j, i_0) + G_0(i, i_0) + G_0(j, j_0)]^2}{4[R - R_0(i_0, j_0)]}. \quad \text{(B2)}$$

Now, taking the limit of Equation (B2) as i or j goes to infinity and using Equation (A7). Thus, we obtain:

$$R(i, j) = R_o(i, j) + \frac{\text{zero}}{4[R - R_0(i_0, j_0)]}. \quad \text{(B3)}$$

Finally, using Equation (A8) and Equation (B3), one gets:

$$R(i, j) = R_0(i, j) \to G_0(0, 0, 0). \quad \text{(B4)}$$

Thus, we conclude that as the separation between sites i and j goes to infinity then, the perturbed resistance goes to the perfect resistance (i.e. it goes to a finite value).

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