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Finite element analysis of internally balanced elastic materials

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Highlights

- Usual hyperelastic framework is augmented with additional tensor field variable arising from multiplicative decomposition of the deformation gradient.
- Energy minimization with respect to the tensor field internal variable generates tensor balance requirement.
- Incremental changes of tensor field internal variable naturally condensed into FE treatment.
- FE treatment resolves singular slipping surfaces that emerge only when load thresholds are exceeded.

Abstract

This paper provides a finite element treatment for a generalized hyperelastic theory in which the deformation gradient is subject to the multiplicative two-factor decomposition that is commonly used in large strain plasticity. The multiplicative decomposition in the present context is motivated by a theory of balanced internal variables, one of which is the first factor in the decomposition. Previous studies have shown how the associated continuum mechanical formulation then leads not only to the stress equations of equilibrium but also to an additional tensor balance equation that serves to determine the individual factors in the decomposition. Such a theory has been shown to predict loading thresholds associated with the emergence of slip surfaces at grip locations in well-posed boundary value problems. However, previous numerical treatments focused on shooting method procedures that failed to resolve loading plateaus and other key features associated with the emergence of such singular surfaces. In the present work we describe a finite element treatment that resolves such previously obscure details. Additional finite element simulations illustrate

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