Finite Element Formulation of Internally Balanced Blatz–Ko Material Model

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Material constitutive models often include internal variables in order to capture realistic mechanical effects such as plasticity and swelling. The implementation of these material models in the frame of nonlinear finite element is well established. Merely, these constitutive models are expressed in terms of deformation gradient $\mathbf{F}$, its derived quantities, and its various multiplicative decompositions such as

$$\mathbf{F} = \hat{\mathbf{F}}\,\breve{\mathbf{F}} \tag{1}$$

The usual treatment is that $\hat{\mathbf{F}}$ models elastic response and it is associated to the rules of variational calculus. The $\breve{\mathbf{F}}$ portion then models inelastic response usually by means of a time dependent evolution law. This multiplicative decomposition serves to pertain particular potions of $\mathbf{F}$ to specific parts of the material response.

A new scheme of viewing hyperelastic material response is introduced in [1]. Actually, the arguments of variational calculus are applied to both portions of the deformation gradient decomposition. The decomposition itself is determined on the basis of an additional internal balance equation that emerges naturally from the variational treatment. The total Lagrange formulation of this new treatment is derived by linearizing the achieved weak form with respect to both portions of multiplicative decomposition [2]. In this work Blatz–Ko material model is generalized to incorporate the notion of internal balance. The internal balance equation is written in terms of the principal stretches that simplifies the procedure of solving it. The internally balanced Blatz–Ko model gives rise to an overall softening response as is demonstrated in the context of homogeneous deformations.

References

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[2] A. Hadoush, H. Demirkoparan, and T. J. Pence. Finite element analysis of internally balanced elastic materials. *Computer Methods in Applied Mechanics and Engineering*, 322:373–395, 2017.