RESISTANCE CALCULATION OF AN INFINITE NETWORK OF RESISTORS- APPLICATION ON GREEN'S FUNCTION

By

Jihad Hasan Jabali Asad

Supervisor

Dr. Jamil Mahmoud Khalifeh, Prof

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University of Jordan

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Examination Committee	**Signature**
Dr. Jamil Khalifeh, Chairman Prof. of Theoretical Solid State Physics	.....
Dr. Marouf H. Abdallah, member Prof. of Molecular Spectroscopy	.....
Dr. Dia Eddin Arafah, member Prof. of Solid State Physics	.....
Dr. Mahmoud A. Hasan, member Prof. of Theoretical Nuclear Physics	.....
Dr. Mahmoud Ismail Jaghoub, **member** Assoc. Prof. of Theoretical Nuclear Physics Hashemite University-Dep. of Physics	.....

Dedication

To my Parents,

Brothers,

Children, and to

My Wife.

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List of Abbreviations

LGF	Lattice Green's Function
SC	Simple Cubic
BCC	Body Centered Cubic
FCC	Face Centered Cubic
TBH	Tight Binding Hamiltonian
BZ	Brillouin Zone

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Abstract

Resistance Calculation of an Infinite Network of Resistors- Application on Green's Functions

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The resistance of an infinite network of identical resistors is calculated in two- and three-dimensions, using the Lattice Green's function (LGF). This work deals with two cases: the perfect lattice and the perturbed lattice (i.e. a bond between two lattice points is removed).

It is shown how to derive the basic formula which relates the resistance to the LGF. In calculating the resistance we make use of the values of the LGF at arbitrary sites and we use some recurrence formulae. Comparison of calculated values is carried out with experimental results for finite square and simple cubic lattices. The asymptotic behavior of the resistance in a square and simple cubic (SC) lattices for both the perfect and perturbed cases is studied.

The study resulted in finding that for a perfect lattice (i.e. square or SC) the resistance is symmetric along the low-index directions, whereas for the perturbed case the symmetry is broken. We demonstrate that the resistance in a square lattice diverges as the separation between the sites increases, while in the SC lattice it tends to a finite

value. Finally, the measured bulk values are in good agreement with those calculated, but as approaching the edge or the surface of the lattice the measured values exceed those calculated.

CHAPTER ONE

GENERAL INTRODUCTION

1.1 Introduction

The Lattice Green's Function (LGF) is a basic function in the study of the solid state physics and condensed matter. It appears especially when impure solids are studied [Morita and Horiguchi, 1972]. Green was the first physicist who established the basic concepts of Green's function in the potential theory, and his work was focused on solving Laplace's and Poisson's equations with different boundary conditions. The use of Green's function method plays an important role in many-body problems [Fetter and Walecka, 1971], especially in problems of solid state physics where an enormous progress has been realized. In the mathematical problem of quantum theory which consists of solving linear operator equations with given boundary conditions, Green's functions constitute the natural language to study boundary conditions.

Nowadays, Green's function is one of the most important concepts in many branches of physics, as many quantities in solid state physics can be expressed in terms of LGF. In the following are some examples: statistical model of ferromagnetism such as Ising model [McCoy and Wu, 1978], Heisenberg model [Dalton and Wood, 1967], spherical model [Lax, 1952], random walk theory [Montrol et. al, 1965], [Hughes, 1986], diffusion [Montet, 1973], band structure [Koster and Slater, 1954], and resistance calculation for an infinite network of identical resistors [Cserti, 2000], [Cserti et. al, 2002].

The LGF for several structure lattices has been widely studied during the second half of the last century. The LGF for the rectangular lattice has been investigated by [Katsura and Inawashiro, 1971], they used the Mellin-Barnes type integral. Recurrence relation, which gives the LGF along the diagonal direction from a couple of values of complete elliptic integrals of the first and second kinds for the rectangular and square lattices, has been derived by [Morita, 1971b].

The LGF for Simple Cubic (SC) lattice at the origin $G(0,0,0)$ has been investigated by many authors: [Joyce, 1973] expressed $G(0,0,0)$ in terms of the complete elliptic integrals of the first kind, [Horiguchi, 1971] expressed $G(1,0,0)$ as a sum of simple integrals of the complete elliptic integrals of the first kind and evaluated it numerically, [Katsura et al., 1971] investigated the LGF for the SC lattice using the Mellin-Barnes type integral. Recently, [Glasser and Boersma, 2000] showed that $G(l,m,n)$ can be expressed rationally in terms of $G(0,0,0)$.

The first attempts to study the LGF for the Body Centered Cubic (BCC) lattice have been carried out by [Maradudin et al., 1960]. They showed that the LGF for the BCC lattice at the origin $G(0,0,0)$ can be expressed as a product of complete elliptic integrals of the first kind. One can find other useful investigations for the LGF of the BCC lattice in many references as [Joyce, 1971a and b and Inoue, 1975].

The LGF for the Face Centered Cubic (FCC) lattice was studied well by [Iwata, 1969], he expressed $G(0,0,0)$ in a compact form as a product of complete elliptic integrals of the first kind. The LGF at any lattice site $G(l,m,n)$ was studied by [Mano, 1974 and Joyce, 1971c]; $G(l,m,n)$ is expressed in terms of linear combinations of complete elliptic integrals of the first and second kind. In their paper [Glasser and Boersma, 2000] expressed the LGF for FCC lattice in terms of the known value of $G(0,0,0)$.

Finally, [Zeitoun, 1991] studied the LGF for the SC lattice at any arbitrary site with the aid of the recurrence formulae and difference equations by using the Mellin-Barnes type integrals. Also, [Sakaji, 1994, Sakaji et al., 2002, a, b, Hijjawi, 2002 and Hijjawi and Khalifeh, 2002] studied the LGF for different dimensions where they evaluated analytically and numerically Green's function, density of states, phase shift

and scattering cross section for one, and two- dimensional lattices, Glasser cubic lattice and BCC lattice.

1.2 Previous Studies

A classic problem in electric circuit theory studied by many authors over many years is computation of the resistance between two nodes in a resistor network. Besides being a central problem in electric circuit theory, the computation of resistances is also relevant to a wide range of problems ranging from random walk [Doyle and Snell, 1984] and [Lovãsz, 1996], theory of harmonic functions [Van der Pol, 1959] to first-passage processes [Render, 2001] to LGF [Katsura and Horiguchi, 1971].

The connection with these problems originates from the fact that electrical potentials on a grid are governed by the same difference equations as those occurring in the other problems. For this reason, the resistance problem is often studied from the point of view of solving the difference equations, which is most conveniently carried out for infinite networks.

Kirchhoff [1847] formulated the study of electric networks more than 150 years ago. The electric- circuit theory is discussed in detail by [Van der Pol and Bremmer, 1955] they derived the resistance between nearby points on the square lattice. Francis J. Bartis [1966] introduced how complex systems can be treated at the elementary level and showed how to calculate the effective resistance between adjacent nodes of a square, triangular, honeycomb and kagome lattices of one-ohm resistors.

Venezian [1994] showed that the resistance between adjacent sites on an infinite square grid of equal resistors can easily be found by the superposition of current distribution; and the mathematical problem involves the solution of an infinite set of linear, inhomogeneous difference equations which are solved by the method of separation of variables. Numerical results for the resistances between the sites $(0,0)$ and

(l,m) in units of R are presented. Atkinson and Van Steenwijk [1998] calculated the resistance between two arbitrary sites in an infinite square lattice of identical resistors. Their method is generalized to infinite triangular- and hexagonal- lattices in two dimensions, and also to infinite cubic and hypercubic- lattices in three and more dimensions.

Monwhea Jeng [1999] introduced a mapping between random walk problems and resistor network problems, where his method was used to calculate the effective resistance between any two sites in an infinite two-dimensional square lattice of unit resistors and the superposition principle was used to find the effective resistances on toroidal- and cylindrical- square- lattices.

Recently, [Cserti, 2000] introduced an alternative method based on the LGF rather than using the superposition distribution of current, where the resistance for d-dimensional hypercubic- rectangular- triangular- and honeycomb- lattices of resistors is discussed in detail. Recurrence formulae for the resistance between arbitrary lattice points of the square lattice have been given in his paper. Cserti's method can be applied in a straightforward manner to other types of lattice structures and can be useful didactically for introducing many concepts used in condensed matter physics.

The resistance between arbitrary nodes of infinite networks of resistors is studied when the network is perturbed by removing one bond from the perfect lattice [Cserti et al., 2002], where the resistance in a perturbed lattice is expressed in terms of the resistance in a perfect lattice.

Finally, [Wu, 2004] obtained the resistance between arbitrary two nodes in a resistor network in terms of the eigenvalues and eigenfunctions of the Laplacian matrix associated with the network. Explicit formulae for two point resistances are deduced in

his paper for regular lattices in one, two, and three- dimensions under various boundary conditions.

1.3 **Thesis Plan**

The plan of this thesis is as follows:

Chapter two is devoted to the general formalism, which includes the derivation of the formulae that relate the resistance in pure- and perturbed- infinite networks of identical resistors to the LGF of the tight-binding Hamiltonian (TBH). Chapter three is concerned with the application of the LGF in calculating the resistance for pure- and perturbed- square lattices, and comparing with experimental results. Chapter four contains the application of the LGF for the pure- and perturbed SC lattices and comparing with experimental results. In chapter five, results and discussion of this work are presented including comparison between experimental measurements and theoretical calculations. Finally, in chapter six a general summary is presented with a partial list of some open problems that can be investigated in future.

CHAPTER TWO

FORMALISM

In this chapter formalism of the problem is presented as follows: the perfect case (section 2.1), the perturbed case (section 2.2), and finally a summary follows in section 2.3.

2.1 **Pure (Perfect) Lattice**

Consider a d-dimensional lattice such that all the lattice points are specified by the position vector $\vec{r}$

$$\vec{r} = l_1\vec{a}_1 + l_2\vec{a}_2 + ... + l_d\vec{a}_d \,.$$

(2.1)

Where $l_1, l_2, ..., l_d$ are integers (positive, negative or zero),

and $\vec{a}_1, \vec{a}_2, ..., \vec{a}_d$ are independent primitive translation vectors.

When all $\vec{a}_i$'s have the same magnitude (i.e. $|\vec{a}_1| = |\vec{a}_2| = ... = |\vec{a}_d| = a$), then the d-dimensional lattice is called a hypercube.

In the case of network of resistors we assume the hypercube to consist of identical resistors (i.e. the same resistance R). In this section we present the resistance between the origin and a given lattice point $\vec{r}$ of the infinite hypercube. To do this let us assume that a current $(+I)$ enters at the origin and a current $(-I)$ exits at a lattice point $\vec{r}$, and zero otherwise. Thus

$$I(\vec{r}\,') = \begin{cases} +\ I\ , & \vec{r}\,' = 0 \\ -\ I\ , & \vec{r}\,' = \vec{r} \\ 0\ , & \text{otherwise.} \end{cases}$$

(2.2)

The above equation can be rewritten as

$$I(\vec{r}\,') = I[\delta_{\vec{r}',0} - \delta_{\vec{r}',\vec{r}}] \,.$$

(2.3)

Also the potential at the lattice point $\vec{r}\,'$ to be $V(\vec{r}\,')$.

According to Ohm's and Kirchhoff's laws we can write

$$I(\vec{r}\,')R_o(\vec{r}\,') = \sum_n [V(\vec{r}\,') - V(\vec{r}\,' + \vec{n})].$$

(2.4)

Where $\vec{n}$ are vectors from site $\vec{r}\,'$ to its nearest neighbors (i.e. $\vec{n} = \pm\vec{a}_i, i = 1,2,...,d$).

Using the so-called lattice laplacian defined on the hypercubic lattice [Cserti, 200] i.e.

$$\Delta_{(\vec{r}\,')} f(\vec{r}) = \sum_n [f(\vec{r} + \vec{n}) - f(\vec{r})] .$$

(2.5)

The right hand side of Eq. (2.4) can be written as:

$$\sum_n [V(\vec{r} + \vec{n}) - V(\vec{r})] = -\Delta_{(\vec{r})} V(\vec{r}) . \tag{2.6}$$

So Eq. (2.4) becomes

$$\Delta_{(\vec{r}')} V(\vec{r}\,') = -I(\vec{r}\,')R_o(\vec{r}\,') .$$

(2.7)

Now, using Eq. (2.3) then $I(\vec{r}\,')R_o(\vec{r}\,')$ can be written as

$$I(\vec{r}\,')R_o(\vec{r}\,') = IR_o(\vec{r}\,')[\delta_{\vec{r}',0} - \delta_{\vec{r}',\vec{r}}];$$

$$= IR_o(\vec{r})[0-1];$$

$$= -IR_o(\vec{r}) .$$

(2.8)

Also, $\Delta_{(\vec{r})} V(\vec{r}) = V(\vec{r}) - V(0)$. So Eq. (2.7) becomes

$$V(\vec{r}) - V(0) = -IR_o(\vec{r});$$

Or

$$R_o(\vec{r}) = \frac{V(0) - V(\vec{r})}{I} . \tag{2.9}$$

To find the resistance we need to solve Eq. (2.7), which is a Poisson-like equation and it may be solved using the LGF, so one may write (comparing with Poisson's equation)

$$V(\vec{r}) = R\sum_{\vec{r}'} G_o(\vec{r} - \vec{r}')I(\vec{r}').$$

(2.10)

where the LGF is defined by

$$\Delta_{(\vec{r}')} G_o(\vec{r} - \vec{r}') = -\delta_{\vec{r}',\vec{r}}.$$

(2.11)

Using Eq. (2.3) and Eq. (2.10) then $V(0)$ and $V(\vec{r})$ can be written as

$$V(0) = IR[G_o(0) - G_o(\vec{r}')].$$

(2.12)

and

$$V(\vec{r}) = IR[G_o(\vec{r}) - G_o(0)].$$

(2.13)

Now, using Eq. (2.9) then

$$R_o(\vec{r}) = \frac{V(0) - V(\vec{r})}{I} = R[G_o(0) - G_o(\vec{r}) - G_o(\vec{r}) + G_o(0)];$$

$$R_o(\vec{r}) = 2R[G_o(0) - G_o(\vec{r})].$$

(2.14)

The last expression is our basic result for the resistance. Once we know the LGF it is easy to obtain the resistance $R_o(\vec{r})$ for a perfect lattice structure.

To find the LGF defined in Eq. (2.11), we take the periodic boundary conditions at the edges of the hypercube, and to do this consider a hypercube with L lattice points along each side; then the number of sites in the d-dimensional hypercube is L^d.

Substituting the Fourier transform

$$G_o(\vec{r}) = \frac{1}{L^d} \sum_{\vec{K} \in BZ} G_o(\vec{K}) \exp(i\vec{K}\vec{r}) .$$

(2.15)

of the LGF into Eq. (2.11). Thus

$$\sum_{\vec{K} \in BZ} \Delta_{(\vec{r}')} \frac{1}{L^d} G_o(\vec{K}) \exp\{i\vec{K}(\vec{r} - \vec{r}')\} = \frac{1}{L^d} \sum_{K \in BZ} G_o(\vec{K}) \sum_{\vec{n}} [\exp\{i\vec{K}(\vec{r} + \vec{n} - \vec{r}')\} - \exp\{i\vec{K}(\vec{r} - \vec{r}')\}] = -\delta_{\vec{r},\vec{r}'}$$

$$\frac{1}{L^d} \sum_{\vec{K} \in BZ} \sum_{\vec{n}} G_o(\vec{K}) [\exp(i\vec{K}\vec{n}) - 1] = -1 ;$$

$$\sum_{\vec{K} \in BZ} \sum_{\vec{n}} G_o(\vec{K}) [\exp(i\vec{K}\vec{n}) - 1] = -\sum_{\vec{K} \in BZ}^{L^d} 1 ;$$

$$G_o(\vec{K}) \sum_{\vec{n}} [\exp(i\vec{K}\vec{n}) - 1] = -1 ;$$

$$G_o(\vec{K}) = \frac{-1}{\sum_{\vec{n}} [\exp(i\vec{K}\vec{n}) - 1]} ;$$

$$G_o(\vec{K}) = \frac{1}{2\sum_{i=1}^{d} (1 - Cos\vec{K}\vec{a}_i)} ;$$

$$G_o(\vec{K}) = \frac{1}{E(\vec{K})} .$$

(2.16)

where

$$E(\vec{K}) = 2\sum_{i=1}^{d} (1 - Cos\vec{K}\vec{a}_i) .$$

(2.17)

and the wave vector defined in Eq. (2.16) is limited to the first Brillouin Zone and is given by

$$\vec{K} = \frac{m1}{L}\vec{b}_1 + \frac{m2}{L}\vec{b}_2 + ... + \frac{m_d}{L}\vec{b}_d \tag{2.18}$$

where

L is assumed to be even,

and

$m_i's$ are integers such that

$$\frac{-L}{2} \le m_i \le \frac{L}{2} \quad \text{for } i = 1,2,...,d \tag{2.19}$$

$\vec{b}_j$ are the reciprocal lattice vectors defined by

$\vec{a}_i\vec{b}_j = 2\pi\delta_{ij} \quad i, j = 1,2,...,d$.

Substituting Eq. (2.16) into Eq. (2.15), the LGF takes the form

$$G_o(\vec{r}) = \frac{1}{L^d}\sum_{\vec{K}\in BZ}\frac{\exp(i\vec{K}\vec{r})}{E(\vec{K})}$$

(2.20)

Taking the limit as $L \longrightarrow \infty$, the summation over $\vec{K}$ can be changed into integration [Ashcroft and Mermin, 1988], i.e.

$$\frac{1}{L^d}\sum_{\vec{K}\in BZ} \longrightarrow v_o \int_{\vec{K}\in BZ}\frac{d^d\vec{K}}{(2\pi)^d}.$$

(2.21)

where $v_o = a^d$ is the volume of the unit cell of the d-dimensional hypercube.

Using Eq. (2.21), then Eq. (2.20) becomes

$$G_o(\vec{r}) = v_o \int_{\vec{K}\in BZ}\frac{d^d\vec{K}}{(2\pi)^d}\frac{\exp(i\vec{K}\vec{r})}{E(\vec{K})}$$

(2.22)

To find the resistance defined by Eq. (2.14)

$$G_o(0) = v_o \int_{\vec{K}\in BZ} \frac{d^d\vec{K}}{(2\pi)^d} \frac{1}{E(\vec{K})} .$$

(2.23)

and

$$G_o(\vec{r}) = v_o \int_{\vec{K}\in BZ} \frac{d^d\vec{K}}{(2\pi)^d} \frac{\exp(i\vec{K}\vec{r})}{E(\vec{K})} .$$

(2.24)

Thus Eq. (2.14) becomes

$$R_o(\vec{r}) = 2Rv_o \int_{\vec{K}\in BZ} \frac{d^d\vec{K}}{(2\pi)^d} \frac{1-\exp(i\vec{K}\vec{r})}{E(\vec{K})} .$$

(2.25)

The last formula can be simplified if the lattice point is specified by Eq. (2.1) and by using Eq. (2.17). Thus

$$R_o(l_1,l_2,...,l_d) = R\int_{-\pi}^{\pi}\frac{dx_1}{2\pi}...\int_{-\pi}^{\pi}\frac{dx_d}{2\pi}\frac{1-\exp(l_1x_1+l_2x_2+...+l_dx_d)}{\sum_{i=1}^{d}(1-Cosx_i)} .$$

(2.26)

Finally, the LGF for a d-dimensional hypercube can be written as [Economou, 1983]

$$G_o(l_1,l_2,...,l_d) = \int_{-\pi}^{\pi}\frac{dx_1}{2\pi}...\int_{-\pi}^{\pi}\frac{dx_d}{2\pi}\frac{\exp(il_1x_1+il_2x_2+...+il_dx_d)}{2\sum_{i=1}^{d}(1-Cosx_i)} .$$

(2.27)

2.2 **Perturbed Lattice (a bond is removed)**

Consider again a d-dimensional infinite lattice made up of identical resistors, and as in section 2.1 we assume that all the lattice points are specified by Eq. (2.1). Before

starting the formalism of the perturbed lattice, let us review the perfect case presented in section 2.1 using Dirac notation.

As in section 2.1 let the potential at site $\vec{r}_i$ be $V(\vec{r}_i)$ and the current entering at origin to be $(+I)$, and the current exiting at a lattice point $\vec{r}_i$ to be $(-I)$. One can form two state vectors, V and I such that

$$V = \sum_i |i\rangle V_i \quad . \tag{2.28}$$

$$I = \sum_i |i\rangle I_i \quad . \tag{2.29}$$

where

$$V_i = V(\vec{r}_i) \, . \tag{2.30}$$

and

$$I_i = I(\vec{r}_i) \, . \tag{2.31}$$

We assume that $\langle i|k\rangle = \delta_{ik}$ and $\sum_i |i\rangle\langle i| = 1$.(i.e. $|i\rangle$ forms a complete orthonormal set).

Using Eq. (2.28) and Eq. (2.29), then Eq. (2.2) can be written as

$$\sum_j (z\delta_{ij} - \Delta_{ij})\langle j|V = R\langle i|I \tag{2.32}$$

where z is the number of neighbors of each lattice site (i.e. $z = 2d$ for a d- dimensional hypercube lattice).

and

$$\Delta_{kl} = \begin{cases} 1, & \vec{r}_k, \vec{r}_l \quad \text{are nearest neighbors} \end{cases} \tag{2.33}$$

⌊ zero, otherwise

Multiplying both sides of Eq. (2.32) by $|i\rangle$ and summing over i, we obtain

$$L_o V = -RI \tag{2.34}$$

where L_o is the so-called lattice laplacian [Cserti et. Al, 2002]

$$L_o = \sum_{i,j} |i\rangle(\Delta_{ij} - z\delta_{ij})\langle i| \tag{2.35}$$

The LGF of the operator L_o is also defined by [Economou, 1983]

$$L_o G_o = -1 \tag{2.36}$$

The solution of Eq. (2.34) is simply

$$V = -RL^{-1}{}_o I \tag{2.37}$$

and from Eq. (2.36)

$L_o^{-1} = -G_o$. Thus Eq. (2.37) becomes

$$V = -RL^{-1}{}_o I = RG_o I \,. \tag{2.38}$$

To measure the resistance between any two arbitrary sites we assume that a current $+I$ enters at $\vec{r}_i$ and $-I$ exits at $\vec{r}_j$, while the current at all other sites is zero, so

$$I_m = I(\delta_{mi} - \delta_{mj}) \qquad \text{for} \qquad \text{all} \qquad m \,. \tag{2.39}$$

Inserting the above relation into Eq. (2.38), one gets

$$\begin{aligned} V_k &= \langle k|V = R\langle k|G_o I \,; \\ &= R\sum_m \langle k|G_o|m\rangle I_m \,; \\ &= RI[G_o(k,i) - G_o(k,j)] \,. \end{aligned} \tag{2.40}$$

Finally, the resistance between sites $\vec{r}_i$ and $\vec{r}_j$ can be written as

$R_o(i,j) = \dfrac{V_i - V_j}{I}$ and using Eq. (2.40), one gets

$$R_o(i,j) = 2R[G_o(i,i) - G_o(i,j)] .$$

(2.41)

Now, let us introduce the formalism of the perturbed lattice (i.e. a bond between the sites $\vec{r}_{io}$ and $\vec{r}_{jo}$ is removed). Again we consider here a d- dimensional infinite lattice made up of identical resistors.

At site $\vec{r}_i$ the current contribution δI_i due to the bond (i_o, j_o) can be written as

$$\delta I_i R = \delta_{ii_o}(V_{i_o} - V_{j_o}) + \delta_{ij_o}(V_{j_o} - V_{i_o});$$

$$= \langle i | i_o \rangle (\langle i_o | - \langle j_o |)V + \langle i | j_o \rangle (\langle j_o | - \langle i_o |)V;$$

$$= \langle i | (| i_o \rangle - | j_o \rangle)(\langle i_o | - \langle j_o |)V;$$

$$\delta I_i R = \langle i | L_1 V .$$

(2.42)

where the operator L_1 is of a so-called "dyadic" form

$$L_1 = (| i_o \rangle - | j_o \rangle)(\langle i_o | - \langle j_o |) . \qquad (2.43)$$

and

$\langle n | m \rangle = \delta_{nm}$ has been used.

Now removing the bond $(i_o j_o)$ from the perfect lattice, then the current I_i at site $\vec{r}_i$ is given by

$$(-L_o V)_i - R\delta I_i = RI_i \qquad (2.44)$$

Thus, Ohm's and Kirchhoff's laws for the perturbed lattice can be written by inserting Eq. (2.42) into Eq. (2.44)

$$LV = -RI \,. \tag{2.45}$$

where $L = L_o + L_1$

Note that the operator L is now a sum of L_o associated with the perfect lattice and a perturbation given by L_1 [Kirkpatrick, 1973].

The LGF for the operator L_1 is given by [Economou, 1983]

$$LG = -1 \,. \tag{2.46}$$

To measure the resistance between sites $\vec{r}_i$ and $\vec{r}_j$ we assume that the current distribution is given by Eq. (2.39).

Using Eq. (2.44) and Eq. (2.46), one can write

$$V = -RL^{-1} I \,;$$

$$= RGI \,. \tag{2.47}$$

So

$$V_k = \langle k | V = R \langle k | GI = R \sum_m \langle k | G | m \rangle I_m \,. \tag{2.48}$$

Substituting Eq. (2.39) into the last expression one gets

$$V_k = IR \langle k | G | m \rangle (\delta_{mi} - \delta_{mj}) \,;$$

$$V_k = IR[G(k,i) - G(k,j)] \,. \tag{2.49}$$

Thus, the resistance between sites $\vec{r}_i$ and $\vec{r}_j$ can be written as

$$R(i,j) = \frac{V_i - V_j}{I} \tag{2.50}$$

From Eq. (2.49)

$$R(i,j) = R[G(i,i) - G(i,j) + G(j,j) - G(j,i)]$$

(2.51)

Note that $G(i,i) \neq G(j,j)$, because the translation symmetry is broken in the perturbed lattice. However, as we shall see $G(i,j) = G(j,i)$.

Now, our problem of finding the resistance reduces to the calculation of the LGF for the perturbed lattice.

Using Eq. (2.35) and Eq. (2.46), one can write

$$LG = -1 \Rightarrow (L_o + L_1)G = -1.$$

(2.52)

$L_o = -G^{-1}{}_o$. Thus the above relation becomes

$$(-G^{-1}{}_o + L_1)G = -1.$$

(2.53)

Multiplying the last relation from left by G_o, one gets

$$-G + G_o L_1 G = -G_o.$$

(2.54)

Finally,

$$G = G_o + G_o L_1 G.$$

(2.55)

Equation (2.55) is called Dyson's equation, which is an equation for G in terms of G_o (which is assumed to be known), and the perturbation L_1. Its solution can be found by the iteration method

$$G = G_o + G_o L_1 G_o + G_o L_1 G_o L_1 G_o + ... \qquad (2.56)$$

If L_1 has a special form as Eq. (2.43), then Dyson's equation can be solved exactly.

Since $L_1 = (|i_o\rangle - |j_o\rangle)(\langle i_o| - \langle j_o|)$, so we may apply the identity

$$(A + |x\rangle\langle y|)^{-1} = A^{-1} - \frac{A^{-1}|x\rangle\langle y|A^{-1}}{1 + \langle y|A^{-1}|x\rangle}. \tag{2.57}$$

The above identity is valid for arbitrary vectors $|x\rangle$ and $|y\rangle$ whose dimensions are the same as the operator A, assuming the inverse of A i.e. A^{-1} exists and $1 + \langle y|A^{-1}|x\rangle \neq 0$. [Cserti et. al, 2002].

Using the above identity with $A = L_o$, $|x\rangle = |i_o\rangle - |j_o\rangle$ and $\langle y| = \langle i_o| - \langle j_o|$. One obtains for the LGF

$$G = -(L_o + L_1)^{-1};$$

$$= -L^{-1}{}_o + \frac{L^{-1}{}_o L_1 L^{-1}{}_o}{1 + (\langle i_o| - \langle j_o|)L^{-1}{}_o(|i_o\rangle - |j_o\rangle)};$$

$$= G_o + \frac{G_o(|i_o\rangle - |j_o\rangle)(\langle i_o| - \langle j_o|)G_o}{1 - [\langle i_o|G_o|i_o\rangle - \langle i_o|G_o|j_o\rangle - \langle j_o|G_o|i_o\rangle + \langle j_o|G_o|j_o\rangle]};$$

$$= G_o + \frac{G_o(|i_o\rangle - |j_o\rangle)(\langle i_o| - \langle j_o|)G_o}{1 - 2[G(i_o, i_o) - G(i_o, j_o)]}.$$

(2.58)

where we have used

$$\langle n|G_o|m\rangle = G_o(n,m),\ G_o(n,m) = G_o(m,n)$$

and

$$G_o(n,n) = G_o(m,m).$$

Note that the denominator $1 - 2[G_o(i_o, i_o) - G_o(i_o, j_o)]$ never equal to zero for $d \rangle 1$.

Expressing G with the matrix elements of G_o, (i.e. taking $\langle i|G|j\rangle = G(i,j)$), then Eq. (2.58) becomes

$$G(i,j) = \langle i|G|\, j\rangle = G_o(i,j) + \frac{[G_o(i,i_o) - G_o(i,j_o)][G_o(i_o,j) - G_o(j_o,j)]}{1 - 2[G_o(i_o,i_o) - G_o(j_o,j_o)]}.$$

(2.59)

There is an alternative way to obtain Eq. (2.59). By inserting L_1 given by Eq. (2.43) into Eq. (2.56).

It is clear from the $G_o(i,j) = G_o(j,i)$, that $G(i,j)$ is also symmetric, (i.e. $G(i,j) = G(j,i)$).

The resistance between i and j can be obtained by Eq. (2.51) and Eq. (2.59)

$$\frac{R(i,j)}{R} = G(i,i) + G(j,j) - 2G(i,j)$$

after some lengthy but straight-forward algebra, one gets

$$\frac{R(i,j)}{R} = R_o(i,j) + \frac{[R_o(i,j_o) + R_o(j,i_o) - R_o(i,i_o) - R_o(j,j_o)]^2}{4[1 - R_o(i_o,j_o)]} \qquad (2.60)$$

Eq. (2.60) is our final result for the resistance between arbitrary nodes i and j of the perturbed lattice in which the bond $(i_o j_o)$ is removed.

It is easy to calculate the resistance between sites i_o and j_o for a d- dimensional hyper cubic lattice. For symmetry reasons [Cserti, 2000] the resistance between i_o and j_o in a perfect lattice is $R_o(i_o,j_o) = \frac{R}{d}$, and then from Eq. (2.60) the resistance between the two ends of the removed bond is $R(i_o,j_o) = \frac{R}{d-1}$ [Cserti et.al, 2002].

2.3 **Summary**

It has been shown in section 2.1 that for the perfect lattice the resistance between the origin and the node $\vec{r}_o$ can be calculated using Eq. (2.15) or using the integral defined by Eq. (2.26). From the final expression of the resistance Eq. (2.26) one can see

that the resistance does not depend on the angles between the unit vectors $\vec{a}_1, \vec{a}_2, ..., \vec{a}_d$. Physically, this means that the hypercube can be deformed without changing the resistance between any two lattice points.

For the perturbed lattice, we show that the resistance between any two arbitrary nodes $\vec{r}_i$ and $\vec{r}_j$ can be obtained either in terms of the perfect LGF or in terms of the perfect resistance. In the derivation of Eq. (2.60) the definition of the lattice Laplacian L_o is not used. Thus, our final result for the resistance Eq. (2.60) in the perturbed lattice is valid for any lattice structure in which each unit cell has only one lattice site.(e.g. square lattice, triangular lattice and simple cubic lattice).

When more than one bond is removed from the perfect lattice, our method can be iterated and lattices with more complex defects can be studied analytically. For example, the so-called crack-type defects arising in several fields such as: electrical and mechanical breakdown phenomena in insulators, thin films and modern ceramics.[Duxbury et.al, 1987 and Boksiner et.al, 1998].

APPLICATIONS:

MAIN RESULTS

CHAPTER THREE

SQUARE LATTICE

In this chapter application of chapter two for a square lattice is presented, in section 3.1 in which we calculate the resistance of the perfect square lattice. In section 3.2 we calculate the resistance of the perturbed square lattice. Finally, in section 3.3 the experimental results for the perfect and perturbed square lattice are presented.

3.1 **Perfect (Pure) Square Lattice**

The resistance in two dimensions between the origin and $\vec{r} = l\vec{a}_1 + m\vec{a}_2$ can be obtained from Eq. (2.26), with $d = 2$

$$R_o(l,m) = R\int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\frac{1-\exp i(lx+my)}{2-Cosx-Cosy}\ ;$$

$$= R\int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\frac{1-[Cos(lx+my)+iSin(lx+my)]}{2-Cosx-Cosy}.$$

(3.1)

Since $\int_{-\pi}^{\pi} Sinxdx = 0$, thus the last expression becomes

$$R_o(l,m) = R\int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\frac{1-Cos(lx+my)}{2-Cosx-Cosy}. \qquad (3.2)$$

Also, the energy dependent LGF of the TBH for a square lattice is given by [Economou, 1983]

$$G_o(E;l,m) = \int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\frac{Cos(lx+my)}{E-Cosx-Cosy}.$$

(3.3)

The last formula is a generalization of our LGF by introducing a variable E instead of 2 in the denominator in Eq. (2.27) for $d = 2$.

Note that a factor 2 appearing in the denominator of Eq. (2.27) is missing in Eq. (3.3). This is related to the fact that in the Schrodinger equation the Laplacian is multiplied by a factor $\frac{1}{2}$ while in our case the Laplacian equation is solved.

To obtain the resistance between the origin and a point $\vec{r}_o = l\vec{a}_1 + m\vec{a}_2$, from Eq. (2.26) for $d = 2$ one obtains

$$R_o(l,m) = R\{\int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\frac{1}{2-Cosx-Cosy} - \int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{Cos(lx+my)}{2-Cosx-Cosy}\} \quad .$$

(3.4)

Comparing the last equation with that given in Eq. (2.27). Thus

$$R_o(l,m) = R[G_o(0,0) - G_o(l,m)] .$$

(3.5)

One can calculate the resistance using Eq. (3.5). The resistance between two adjacent sites (i.e. (1, 0)), is

$$R_o(1,0) = R[G_o(0,0) - G_o(1,0)] \qquad (3.6)$$

$G_o(1,0)$ Can be expressed as [Hijjawi, 2002]

$$G_o(1,0) = \frac{1}{2}[EG_o(0,0) - 1]$$

Substituting the last expression into Eq. (3.5), one gets

$$R_o(1,0) = R[G_o(0,0) - \frac{E}{2}G_o(0,0) + \frac{1}{2}] \qquad ; \; E = 2$$

$$= R[G_o(0,0) - G_o(0,0) + \frac{1}{2}];$$

$$= \frac{R}{2} . \qquad (3.7)$$

Since, $R_o(l,m) = R_o(m,l)$, i.e. due to the symmetry of the lattice, then $R_o(1,0) = R_o(0,1) = \frac{R}{2}$. The same result was obtained by [Cserti, 2002], [Venezian, 1994] and [Aitchison, 1964].

To calculate the resistance between the origin and the second nearest neighbors (i.e. (1, 1)) is

$$R_o(1,1) = R[G_o(0,0) - G_o(1,1)]. \tag{3.8}$$

$G_o(1,1)$ can be expressed in terms of $G_o(0,0)$ and $G'_o(0,0)$ as [Hijjawi, 2002]

$$G_o(1,1) = (\frac{t^2}{2} - 1)G_o(0,0) - \frac{t}{2}(4 - t^2)G_o'(0,0);$$

$$G_o(0,0) = \frac{2}{\pi t}K(\frac{2}{t}). \tag{3.9}$$

and

$$G_o'(0,0) = \frac{-E(\frac{2}{t})}{\pi t(t-2)} - \frac{1}{\pi t^2}K(\frac{2}{t}). \tag{3.10}$$

Where

$K(\frac{2}{t})$ and $E(\frac{2}{t})$ are the elliptic integrals of the first kind and second kind respectively, and

t = 2, is the energy.

Substituting the last two expressions into Eq. (3.8), one obtains

$$R_o(1,1) = \frac{2R}{\pi}. \tag{3.11}$$

Again our result is the same as others [Cserti, 2000] and [Venezain, 1994].

Using the previous method one can calculate $R_o(2,0), R_o(2,1), R_o(3,0)$ and so on. Or one can calculate $R_o(l,m)$ using some recurrence formulae based on those derived for the LGF by [Morita, 1975], and those formulae are

$$G_o(E;m+1,m+1) = \frac{4m}{2m+1}\left(\frac{E^2}{2\gamma^2}-1\right)G_o(E;m,m) - \left(\frac{2m-1}{2m+1}\right)G_o(E;m-1,m-1) \qquad (3.12)$$

$$G_o(E;m+1,m) = \frac{2EG_o(E;m,m) - 2\gamma G_o(E;m,m-1)}{2\gamma} \qquad (3.13)$$

$$G_o(E;m+1,0) = \frac{2EG_o(E;m,0) - \gamma G_o(E;m-1,0) - 2\gamma G_o(E;m,1)}{\gamma} \qquad (3.14)$$

$$G_o(E;m+1,p) = \frac{2EG_o(E;m,p) - \gamma G_o(E;m-1,p) - \gamma G_o(E;m,p+1) - \gamma G_o(Em,p-1)}{\gamma}$$

$$\text{for } 0\langle p\langle m \qquad (3.15)$$

where $\gamma = 1$ and $E = 2$.

Substituting the last four equations into Eq. (3.5), one gets the following recurrence formulae for the resistance

$$R_o(m+1,m+1) = \frac{4m}{2m+1}R_o(m,m) - \frac{2m-1}{2m+1}R_o(m-1,m-1)$$

(3.16)

$$R_o(m+1,m) = 2R_o(m,m) - R_o(m,m-1)$$

(3.17)

$$R_o(m+1,0) = 4R_o(m,0) - R_o(m-1,0) - 2R_o(m,1)$$

(3.18)

$$R_o(m+1,p) = 4R_o(m,p) - R_o(m-1,p) - R_o(m,p+1) - R_o(m,p-1) \qquad (3.19)$$

$$\text{for } 0\langle p\langle m.$$

Using the last four recurrence formulas and the values of $R_o(1,0)$ and $R_o(1,1)$ with the trivial one $R_o(0,0)=0$, we can calculate easily the resistance exactly between the origin and any other site. As an examples

1-Take m=1 and use Eq. (3.16). Thus

$$R_o(2,2)=\frac{4}{3}R_o(1,1)-\frac{1}{3}R_o(0,0)$$

$$=\frac{8}{3\pi}R-0=\frac{8}{3\pi}R=0.848826R\,.$$

2-Take m=1 and use Eq. (3.17). Thus

$$R_o(2,1)=2R_o(1,1)-R_o(1,0)$$

$$=\frac{4}{\pi}R-\frac{1}{2}R=0.773239R$$

3-Take m=1 and use Eq. (3.18). Thus

$$R_o(2,0)=4R_o(1,0)-R_o(0,0)-2R_o(1,1)$$

$$=\frac{4}{2}R-0-\frac{4}{\pi}R=0.726760R\,.$$

4- Take m=2 and use Eq. (3.17). Thus

$$R_o(3,2)=2R_o(2,2)-R_o(2,1)$$

$$=2(0.848826)R-0.773239R$$
$$=0.924413R.$$

5- Take m=2 and use Eq. (3.18). Thus

$$R_o(3,0)=4R_o(2,0)-R_o(1,0)-2R_o(2,1)$$

$$=4(0.726760)R-0.5R-2(0.773239)R$$
$$=0.860562R.$$

And so on for other values, Table 1 shows the values of the resistance between the origin and arbitrary sites for a perfect square lattice. This way we obtain same results as others [Atkinson and Van Steenwijk, 1999].

Table 1: **Calculated and measured values of the resistance between the origin and an arbitrary site in a perfect square lattice.**

The Site (l,m)	$\frac{R_o(l,m)}{R}$ Theoretically	$\frac{R_o(l,m)}{R}$ Experimentally	The Site (l,m)	$\frac{R_o(l,m)}{R}$ Theoretically	$\frac{R_o(l,m)}{R}$ Experimentally
(1,0)	0.5	0.4997	**(1,1)**	0.63662	0.6379
(2,0)	0.72676	0.7283	**(2,2)**	0.84883	0.8527
(3,0)	0.860563	0.8642	**(3,3)**	0.97615	0.9853
(4,0)	0.953987	0.9616	**(4,4)**	1.06710	1.086
(5,0)	1.0258	1.039	**(5,5)**	1.13783	1.169
(6,0)	1.08423	1.104	**(6,6)**	1.19571	1.244
(7,0)	1.13352	1.162	**(7,7)**	1.24468	1.316
(8,0)	1.17616	1.214	**(8,8)**	1.28712	1.388
(9,0)	1.21375	1.263	**(9,9)**	1.32457	1.464
(10,0)	1.24735	1.313	**(10,10)**	1.35807	1.549
(11,0)	1.27774	1.362	**(11,11)**	1.38839	1.648
(12,0)	1.30547	1.416	**(12,12)**	1.41607	1.769
(13,0)	1.33098	1.481	**(13,13)**	1.44153	1.931
(14,0)	1.35459	1.571	**(14,14)**	1.464521	2.177
(15,0)	1.37657	1.755	**(15,15)**	1.486464	2.707
(-1,0)	0.5	0.5011	**(1-,1-)**	0.63662	0.6376
(-2,0)	0.72676	0.7287	**(-2,-2)**	0.84883	0.8525
(-3,0)	0.860563	0.8649	**(-3,-3)**	0.97615	0.9860
(-4,0)	0.953987	0.9622	**(-4,-4)**	1.06710	1.085
(-5,0)	1.0258	1.039	**(-5,-5)**	1.13783	1.169
(-6,0)	1.08423	1.104	**(-6,-6)**	1.19571	1.244
(-7,0)	1.13352	1.161	**(7-,7-)**	1.24468	1.316
(-8,0)	1.17616	1.214	**(-8,-8)**	1.28712	1.388
(-9,0)	1.21375	1.264	**(-9,-9)**	1.32457	1.464
(-10,0)	1.24735	1.313	**(-10,-10)**	1.35807	1.549
(-11,0)	1.27774	1.362	**(-11,-11)**	1.38839	1.648
(-12,0)	1.30547	1.416	**(-12,-12)**	1.41607	1.769
(-13,0)	1.33098	1.481	**(-13,-13)**	1.44153	1.930
(-14,0)	1.35459	1.570	**(-14,-14)**	1.464521	2.177
(-15,0)	1.37657	1.754	**(-15,-15)**	1.486464	2.708

The advantages of the recurrence formulae are that they provide a new, very simple and effective tool to calculate the resistance. Others gave also the exact values of the resistance for nearby points in a square lattice using a different approach [Van der Pol et.al, 1959].

It is important to study the asymptotic behavior of the resistance for large values of l or/and m. To do this we derive first the asymptotic behavior of the LGF for square lattice, [see appendix A];

where the final result is obtained as:

$$G_o(\vec{r}) = G_o(0) - \frac{1}{2\pi}(Ln\frac{|\vec{r}|}{a} + \gamma + \frac{Ln8}{2}).$$

(3.20)

Inserting Eq. (3.20) into the general result of the resistance given in Eq. (2.14). Thus the asymptotic form of the resistance is

$$R_o(l,m) = \frac{R}{\pi}(Ln\sqrt{l^2+m^2} + \gamma + \frac{Ln8}{2})$$

(3.21)

where $\gamma = 0.5772$ is the Euler-Mascheroni constant [Arfken and Weber, 1995]. The resistance is logarithmically divergent for large values of l and m, as shown in Appendix B.

3.2 **Perturbed square lattice (a bond is missing)**

As discussed in section 2.2 one can calculate the resistance between any two arbitrary sites using the last expression derived in Eq. (2.60). It is simple to find the resistance between the ends of the missing bond. On the one hand it is well known [Aitchison, 1964] that for a perfect lattice the resistance between adjacent sites is $\frac{R}{2}$ and on the other hand, this resistance equals the parallel resultant of R and the resistance we wish to find. Thus

$$\frac{1}{\frac{R}{2}} = \frac{1}{R} + \frac{1}{R'}.$$

(3.22)

So, $R' = R$ (i.e. the resistance between the ends of the removed bond) [Cserti et.al].

Now, by noting that $i = (i_x, i_y)$, $j = (j_x, j_y)$, $i_o = (i_{ox}, i_{oy})$ and $j_o = (j_{ox}, j_{oy})$,

one can write equation (2.60) as

$$R(j_x - i_x, j_y - i_y) = R_o(j_x - i_x, j_y - i_y) +$$

$$\frac{[R_o(j_{ox} - i_x, j_{oy} - i_y) + R_o(i_{ox} - j_x, i_{oy} - j_y) - R_o(i_{ox} - i_x, i_{oy} - i_y) - R_o(j_{ox} - j_x, j_{oy} - j_y)]^2}{4[R - R_o(j_{ox} - i_{ox}, j_{oy} - i_{oy})]}$$

(3.23)

To study the asymptotic behavior of the resistance of the perturbed square lattice, substituting Eq. (3.21) into Eq. (3.23). Thus, one obtains

$$R(i, j) = R_o(i, j) + \frac{\left[\frac{R}{\pi} Ln \sqrt{\frac{i^2 j^2 + i^2 {i_o}^2 + {j_o}^2 j^2 + {j_o}^2 {i_o}^2}{i^2 j^2 + i^2 {j_o}^2 + {i_o}^2 j^2 + {i_o}^2 {j_o}^2}}\right]^2}{4[R - R_o(i, j)]}$$

(3.24)

Now, as i or/and j goes to infinity then the limit of the numerator goes to zero. So, one yields that

$$R(i, j) = R_o(i, j)$$

(3.25)

that is, the perturbed resistance between arbitrary sites goes to the perfect resistance as the separation between the two sites goes to infinity. The derivation of Eq. (3.24) is given in details in appendix C.

To calculate the resistance one has to specify the removed bond before starting the calculations and as an example let us consider the removed bond to be $i_o = (0,0)$ and $j_o = (1,0)$. Now it is simple to calculate the resistance between any two arbitrary sites using Eq. (3.23), the above missing bond and the values obtained in section 3.1 for the resistance of the perfect lattice. Our results are arranged in Table 2, and below are some examples:

1-The resistance between the two ends of the missing bond. (i.e. $i = (0,0)$ and $j = (1,0)$).

$$R(1,0) = R_o(1,0) + \frac{[R_o(1,0) + R_o(0,1) - R_o(0,0) - R_o(0,0)]^2}{4[R - R_o(1,0)]}$$

$$R(1,0) = \frac{R}{2} + \frac{[\frac{R}{2} + \frac{R}{2} - 0 - 0]^2}{4[R - \frac{R}{2}]}$$

$= R.$

2-The resistance between $i = (0,0)$ and $j = (2,0)$

$$R(2,0) = R_o(2,0) + \frac{[R_o(1,0) + R_o(2,0) - R_o(0,0) - R_o(1,0)]^2}{4[R - R_o(1,0)]}$$

$$= 0.726760R + \frac{[\frac{R}{2} + 0.726760R - 0 - \frac{R}{2}]^2}{2R} = 0.990850R.$$

3-The resistance between $i = (0,0)$ and $j = (-1,0)$

$$R(-1,0) = R_o(1,0) + \frac{[R_o(1,0) + R_o(1,0) - R_o(0,0) - R_o(2,0)]^2}{4[R - R_o(1,0)]}$$

$= 0.537330R.$

4-The resistance between $i = (0,0)$ and $j = (-2,0)$

$$R(-2,0) = R_o(2,0) + \frac{[R_o(1,0) + R_o(2,0) - R_o(0,0) - R_o(3,0)]^2}{4[R - R_o(1,0)]}$$

$= 0.793810R.$

5- The resistance between $i = (0,0)$ and $j = (-3,0)$

$$R(-3,0) = R_o(3,0) + \frac{[R_o(1,0) + R_o(3,0) - R_o(0,0) - R_o(4,0)]^2}{4[R - R_o(1,0)]}$$

$= 0.94322R.$

Table 2: Calculated and measured values of the resistance between the sites $i = (0,0)$ and $j = (j_x, j_y)$, for a perturbed square lattice (i.e. the bond between $i_o = (0,0)$ and $j_o = (1,0)$ is broken).

The Site $j = (j_x, j_y)$	$\frac{R(i,j)}{R}$ **Theoretically**	$\frac{R(i,j)}{R}$ **Experimentally**	**The Site** $j = (j_x, j_y)$	$\frac{R(i,j)}{R}$ **Theoretically**	$\frac{R(i,j)}{R}$ **Experimentally**
(1,0)	1	1.002	**(0,1)**	0.56602	0.5657
(2,0)	0.99085	0.9939	**(0,2)**	0.82960	0.8305
(3,0)	1.06142	1.067	**(0,3)**	0.97567	0.9793
(4,0)	1.13006	1.141	**(0,4)**	1.07364	1.081
(5,0)	1.18929	1.205	**(0,5)**	1.14747	1.161
(6,0)	1.24015	1.264	**(0,6)**	1.20696	1.227
(7,0)	1.28438	1.317	**(0,7)**	1.25687	1.285
(8,0)	1.32339	1.366	**(0,8)**	1.29990	1.338
(9,0)	1.35825	1.414	**(0,9)**	1.33776	1.387
(10,0)	1.38971	1.461	**(0,10)**	1.37155	1.437
(11,0)	1.41840	1.510	**(0,11)**	1..40208	1.487
(12,0)	1.44472	1.564	**(0,12)**	1.42992	1.541
(13,0)	1.46906	1.628	**(0,13)**	1.45551	1.606
(14,0)	1.49167	1.717	**(0,14)**	1.47919	1.696
(15,0)	1.51280	1.900	**(0,15)**	1.50122	1.879
(-1,0)	0.53733	0.5384	**(0,-1)**	0.56602	0.5674
(-2,0)	0.79381	0.7953	**(0,-2)**	0.82960	0.8310
(-3,0)	0.94322	0.9464	**(0,-3)**	0.97567	0.9802
(-4,0)	1.04566	1.052	**(0,-4)**	1.07364	1.082
(-5,0)	1.12329	1.135	**(0,-5)**	1.14747	1.161
(-6,0)	1.18580	1.203	**(0,-6)**	1.20696	1.227
(-7,0)	1.23811	1.263	**(0,-7)**	1.25687	1.285

(-8,0)	1.28307	1.317	**(0,-8)**	1.29990	1.338
(-9,0)	1.32251	1.369	**(0,-9)**	1.33776	1.388
(-10,0)	1.35762	1.418	**(0,-10)**	1.37155	1.438
(-11,0)	1.38926	1.468	**(0,-11)**	1..40208	1.489
(-12,0)	1.41804	1.523	**(0,-12)**	1.42992	1.545
(-13,0)	1.44445	1.587	**(0,-13)**	1.45551	1.615
(-14,0)	1.46884	1.677	**(0,-14)**	1.47919	1.695
(-15,0)	1.49150	1.860	**(0,-15)**	1.50122	1.915

Now, the broken bond is shifted to be between the sites $i_o = (1,0)$ and $j_o = (2,0)$. Thus, using the above method one calculates the resistance between the origin and any other site. Our results are arranged in Table 3 below.

Table 3: Calculated and measured values of the resistance between the sites $i = (0,0)$ and $j = (j_x, j_y)$, for a perturbed square lattice (i.e. the bond between $i_o = (1,0)$ and $j_o = (2,0)$ is broken).

The Site $j = (j_x, j_y)$	$\frac{R(i,j)}{R}$ **Theoretically**	$\frac{R(i,j)}{R}$ **Experimentally**	**The Site** $j = (j_x, j_y)$	$\frac{R(i,j)}{R}$ **Theoretically**	$\frac{R(i,j)}{R}$ **Experimentally**
(1,0)	0.53733	0.5372	**(0,1)**	0.50406	0.5038
(2,0)	0.99085	0.9939	**(0,2)**	0.73819	0.7398
(3,0)	0.96340	0.9689	**(0,3)**	0.87733	0.881
(4,0)	1.01899	1.029	**(0,4)**	0.97391	0.9816
(5,0)	1.07706	1.092	**(0,5)**	1.04757	1.061
(6,0)	1.12880	1.151	**(0,6)**	1.10712	1.127
(7,0)	1.17419	1.204	**(0,7)**	1.15712	1.185
(8,0)	1.21426	1.255	**(0,8)**	1.20023	1.238
(9,0)	1.25004	1.303	**(0,9)**	1.26257	1.288
(10,0)	1.26214	1.351	**(0,10)**	1.27200	1.337
(11,0)	1.31163	1.400	**(0,11)**	1.30257	1.387
(12,0)	1.33853	1.454	**(0,12)**	1.33044	1.441
(13,0)	1.36336	1.518	**(0,13)**	1.35606	1.506
(14,0)	1.38641	1.606	**(0,14)**	1.37975	1.596
(15,0)	1.40791	1.790	**(0,15)**	1.40180	1.779
(-1,0)	0.50432	0.5058	**(0,-1)**	0.50406	0.5052
(-2,0)	0.73565	0.7377	**(0,-2)**	0.73819	0.7402
(-3,0)	0.87257	0.8766	**(0,-3)**	0.87733	0.8817
(-4,0)	0.96815	0.9753	**(0,-4)**	0.97391	0.9822
(-5,0)	1.04155	1.054	**(0,-5)**	1.04757	1.061
(-6,0)	1.10118	1.120	**(0,-6)**	1.10712	1.127
(-7,0)	1.15141	1.178	**(0,-7)**	1.15712	1.185

(-8,0)	1.19482	1.231	**(0,-8)**	1.20023	1.238
(-9,0)	1.23303	1.282	**(0,-9)**	1.26257	1.288
(-10,0)	1.26716	1.330	**(0,-10)**	1.27200	1.337
(-11,0)	1.29799	1.380	**(0,-11)**	1.30257	1.387
(-12,0)	1.32722	1.434	**(0,-12)**	1.33044	1.441
(-13,0)	1.35195	1.499	**(0,-13)**	1.35606	1.506
(-14,0)	1.37585	1.588	**(0,-14)**	1.37975	1.595
(-15,0)	1.39809	1.772	**(0,-15)**	1.40180	1.779

From the above Tables, one can see that the resistance in the perturbed case is always larger than that in the perfect case. This is due to the positive contribution of the second term in equation Eq. (2.60). The resistance is not symmetric (i.e. $R(l,m) \neq R(m,l)$ because the translational symmetry is broken.

3.3 **Experimental results**

To study the resistance of the square lattice experimentally we constructed a finite square network of identical resistances (R) consisting of (30×30) resistors, each has a value of ($1\ k\Omega$) and a tolerance of (1%) as shown in Fig. 1.

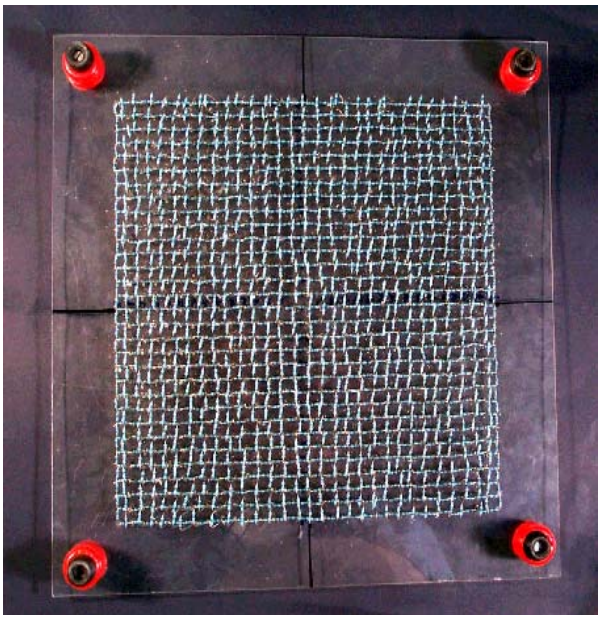

Fig. 1: A square mesh consisting of (30x30) identical resistors.

Using the perfect mesh shown in Fig. 1 above, we measured the resistance between the origin and the site (l, m) along the directions [10], [01], and [11]**.** Our results are arranged in Table 1 above. To measure the resistance for the perturbed case we removed the bond between $i_o = (0,0)$ and $j_o = (1,0)$, then we measured the resistance between the site $i = (0,0)$ and the site $j = (j_x, j_y)$ along the directions [10], [01], and [11]. Our results are arranged in Table 2 above.

Now, the removed bond is shifted, $i_o = (1,0)$ and $j_o = (2,0)$, then we measured again the resistance between the site $i = (0,0)$ and the site $j = (j_x, j_y)$ along the directions [10], [01], and [11]. Our results are arranged in Table 3 above.

CHAPTER FOUR

SIMPLE CUBIC LATTICE

In this chapter application of chapter two for the SC lattice is presented, in section 4.1 in which we calculate the resistance of the perfect SC lattice. In section 4.2 we calculate the resistance of the perturbed SC lattice. Finally, in section 4.3 the experimental results for the perfect and perturbed SC lattice are presented.

4.1 Perfect SC lattice

For a perfect SC lattice, the resistance between the origin and a lattice point $\vec{r}_o = l\vec{a}_1 + m\vec{a}_2 + n\vec{a}_3$ can be obtained from Eq. (2.25) with d=3. Thus

$$R_o(l,m,n) = R\int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\int_{-\pi}^{\pi}\frac{dz}{2\pi}\frac{1-\exp(ilx-imy+inz)}{3-Cosx-Cosy-Cosz} \tag{4.1}$$

$$= R\int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\int_{-\pi}^{\pi}\frac{dz}{2\pi}\frac{1-Cos(lx+my+nz)}{3-Cosx-Cosy-Cosz} \tag{4.2}$$

Similar to the case of a square lattice, the exact values of the resistance between two adjacent lattice sites can be calculated from Eq. (4.2). Because of the symmetry one can write

$$R_o(1,0,0)+R_o(0,1,0)+R_o(0,0,1) = R\int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\int_{-\pi}^{\pi}\frac{dz}{2\pi}[\frac{1-Cosx}{3-Cosx-Cosy-Cosz}$$

$$+\frac{1-Cosy}{3-Cosx-CCosy-Cosz}+\frac{1-Cosz}{3-Cosx-Cosy-Cosz}] = R$$

Therefore the resistance between adjacent sites is $\frac{R}{3}$. In general for a d-dimensional hypercube the resistance between adjacent sites is $\frac{R}{d}$.

The energy dependent LGF of the tight-binding Hamiltonian for a SC lattice is defined as [Economou, 1983]

$$G_o(E;l,m,n) = \int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\int_{-\pi}^{\pi}\frac{dz}{2\pi}\frac{Cos(lx+my+nz)}{E-Cosx-Cosy-Cosz} \tag{4.3}$$

This is a generalization of the LGF by introducing a new variable E instead of the value 3 in the denominator in Eq. (2.27) for $d = 3$. The missing factor of two in the denominator in Eq. (4.3) has been explained in section 3.1.

The resistance between the origin and a lattice site $\vec{r}_o = l\vec{a}_1 + m\vec{a}_2 + n\vec{a}_3$, can be obtained using equation (2.26) with $d = 3$. Thus

$$R_o(l,m,n) = R\int_{-\pi}^{\pi}\frac{dx}{2\pi}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\int_{-\pi}^{\pi}\frac{dz}{2\pi}\left[\frac{1-Cos(lx+my+nz)}{3-Cosx-Cosy-Cosz}\right] \tag{4.4}$$

Compare with Eq. (4.3), for $d = 3$

$$R_o(l,m,n) = R[G_o(3;0,0,0) - G_o(3;l,m,n)] \tag{4.5}$$

The resistance of an infinite SC network of identical resistors between the origin and any lattice site (l,m,n) can be expressed as:

$$\frac{R_0(l,m,n)}{R} = \rho_1 g_0 + \frac{\rho_2}{\pi^2 g_0} + \rho_3 \tag{4.6}$$

where $g_0 = G_0(0,0,0)$ is the LGF at the origin,

and ρ_1, ρ_2, ρ_3 are related to r_1, r_2, r_3 or Duffin and Shelly's parameters $\lambda_1, \lambda_2, \lambda_3$ [Glasser and Boersma, 2000] and [Duffin and Shelly, 1958]) as:

$$\rho_1 = 1 - r_1 = 1 - \lambda_1 - \frac{15}{12}\lambda_2$$

$$\rho_2 = -r_2 = \frac{1}{2}\lambda_2 \tag{4.7}$$

$$\rho_3 = -r_3 = \frac{1}{3}\lambda_3$$

Various values of r_1, r_2, r_3 are shown [Glasser and Boersma, 2000] in Table 1, for (l, m, n) ranging from (0,0,0) to (5,5,5). To obtain other values of r_1, r_2, r_3 one has to use the relation [Horiguchi, 1971]

$$G_0(l+1,m,n) + G_0(l-1,m,n) + G_0(l,m+1,n) + G_0(l,m-1,n) + G_0(l,m,n+1) + G_0(l,m,n-1) = -2\delta_{l0}\delta_{m0}\delta_{n0} + 2EG_0(l,m,n) \tag{4.8}$$

where $E = 3$, is the energy.

In some cases one may use the recurrence formulae Eq. (4.8) two or three times to calculate different values of r_1, r_2, r_3 for (l, m, n) beyond $(5,5,5)$.

Table 4: Values of the resistance in a perfect infinite SC lattice for arbitrary sites.

Site *lmn*	ρ_1	ρ_2	ρ_3	$\frac{R_0(l,m,n)}{R} = \rho_1 g_0 + \frac{\rho_2}{\pi^2 g_0} + \rho_3$
000	0	0	0	0
100	0	0	1/3	0.333333
110	7/12	1/2	0	0.395079
111	9/8	-3/4	0	0.418305
200	-7/3	-2	2	0.419683
210	5/8	9/4	-1/3	0.433598
211	5/3	-2	0	0.441531
220	-37/36	29/6	0	0.449351
221	31/16	-21/8	0	0.453144
222	3/8	27/20	0	0.460159
300	-33/2	-21	13	0.450371
310	115/36	85/6	-4	0.454415
311	15/4	-21/2	2/3	0.457396
320	-271/48	119/8	1/3	0.461311
321	161/36	-269/30	0	0.463146
322	-19/16	213/40	0	0.467174
330	-47/3	1046/25	0	0.468033
331	38/3	-148/5	0	0.469121
332	-26/9	1012/105	0	0.471757
333	51/16	-1587/280	0	0.475023
400	-985/9	-542/3	92	0.464885

410	531/16	879/8	-115/3	0.466418
411	11/2	-357/5	12	0.467723
420	-2111/72	13903/300	6	0.469777
421	245/16	-1251/40	-1	0.470731
422	-32/3	1024/35	0	0.473076
430	-2593/48	28049/200	-1/3	0.473666
431	1541/36	-110851/1050	0	0.474321
432	-493/32	4617/112	0	0.476027
433	667/72	-8809/420	0	0.478288
440	-5989/36	620161/1470	0	0.477378
441	4197/32	-919353/2800	0	0.477814
442	-2927/48	31231/200	0	0.479027
443	571/32	-119271/2800	0	0.480700
444	-69/8	186003/7700	0	0.482570
500	-9275/12	-3005/2	2077/3	0.473263
510	11653/36	138331/150	-348	0.473986
511	-271/4	-5751/10	150	0.474646
520	-2881/16	15123/200	229/3	0.475807
521	949/12	-27059/350	-24	0.476341
522	-501/8	4209/28	2	0.477766
530	-3571/18	1993883/3675	-8	0.478166
531	1337/8	-297981/700	4/3	0.478565
532	-2519/36	187777/1050	0	0.479693
533	2281/48	-164399/1400	0	0.481253
540	-18439/32	28493109/19600	1/3	0.480653
541	1393/3	-286274/245	0	0.480920
542	-7745/32	1715589/2800	0	0.481798
543	5693/72	-4550057/23100	0	0.483012
544	-1123/32	560001/6160	0	0.484441
550	-196937/108	101441689/22050	0	0.483050
551	12031/8	-18569853/4900	0	0.483146
552	-1681/2	5718309/2695	0	0.483878
553	5175/16	-2504541/3080	0	0.484777
554	-24251/312	-1527851/7700	0	0.485921
555	9459/208	-12099711/107800	0	0.487123
600	-34937/6	-313079/25	5454	0.478749
610	71939/24	160009/20	-9355/3	0.479137
633	18552/72	-747654/1155	0	0.483209
644	-388051/1872	23950043/46200	0	0.486209
655	13157/78	-5698667/13475	0	0.488325
700	-553847/12	5281913/50	44505	0.482685

The value of the LGF at the origin was first evaluated by [Watson, 1939] in his famous paper, where he found that

$$G_o(0,0,0) = (\frac{2}{\pi})^2 (18 + 12\sqrt{2} - 10\sqrt{3} - 7\sqrt{6})[K(k_o)]^2 = 0.505462.$$

with $k_o = (2-\sqrt{3})(\sqrt{3}-\sqrt{2})$

and

$K(k) = \int_0^{\frac{\pi}{2}} d\theta \frac{1}{\sqrt{1 - k^2 Sin^2\theta}}$ is the complete elliptic integral of the first kind.

A similar result was obtained by [Glasser and Zucker, 1977] in terms of gamma function.

The asymptotic behavior (i.e. as *l*, or *m*, or $n \to \infty$) of the resistance in a SC is [see Appendix D]

$$\frac{R_o(l,m,n)}{R} \to G_o(0,0,0).$$

4.2 The SC lattice (Perturbed case)

To calculate the resistance between the site $i = (i_x, i_y, i_z)$ and the site $j = (j_x, j_y, j_z)$, one has to specify the removed (missing) bond between $i_o = (i_{ox}, i_{oy}, i_{oz})$ and $j_o = (j_{ox}, j_{oy}, j_{oz})$. Thus the perturbed relation obtained in section 2.2 becomes:

$$R(j_x - i_x, j_y - i_y, j_z - i_z) = R_o(j_x - i_x, j_y - i_y, j_z - i_z) +$$

$$\frac{1}{4[R - R_o(j_{ox} - i_{ox}, j_{oy} - i_{oy}, j_{oz} - i_{oz})]} \{$$

$$R_o(j_{ox} - i_x, j_{oy} - i_y, j_{oz} - i_z) - R_o(i_{ox} - j_x, i_{oy} - j_y, i_{oz} - j_z) - R_o(i_{ox} - i_x, i_{oy} - i_y, i_{oz} - i_z) -$$

$$R_o(j_{ox} - j_x, j_{oy} - j_y, j_{oz} - j_z)\}^2. \qquad (4.9)$$

To study the asymptotic behavior of the resistance of the perturbed SC lattice, substituting Eq. (4.5) into Eq. (4.9). Thus, one obtains

$$R(i,j) = R_o(i,j) + \frac{R}{4[1 - G_o(0,0,0) + G_o(j_{ox} - i_{ox}, j_{oy} - i_{oy}, j_{oz} - i_{oz})]}\{G_o(0,0,0) -$$

$$G_o(j_{ox} - i_x, j_{oy} - i_y, j_{oz} - i_z) + G(0,0,0) - G_o(i_{ox} - j_x, i_{oy} - j_y, i_{oz} - j_z) - G(0,0,0) +$$

$$G_o(i_{ox} - i_x, i_{oy} - i_y, i_{oz} - i_z) - G_o(0,0,0) + G_o(j_{ox} - j_x, j_{oy} - j_y, j_{oz} - j_z)\}^2 .$$

The quantity $G_o(j_{ox} - i_{ox}, j_{oy} - i_{oy}, j_{oz} - i_{oz}) = G_o(1,0,0)$, whatever the broken bond is. Thus, using Eq. (4.5) with $R_o(1,0,0) = \frac{R}{3}$ one gets

$$R(i,j) = R_o(i,j) + \frac{R}{\frac{8}{3}}\{-G_o(j_{ox} - i_x, j_{oy} - i_y, j_{ox} - i_z) - G_o(i_{ox} - j_x, i_{oy} - j_y, i_{oz} - j_z)$$
$$+ G_o(i_{ox} - i_x, i_{oy} - i_y, i_{oz} - i_z) + G_o(j_{ox} - j_x, j_{oy} - j_y, j_{oz} - j_z)\}^2 .$$

Using $G_o(l,m,n) \to 0$ as any of *l, m, n* goes to infinity the second term in the above equation cancels out. So; $R(i,j) = R_o(i,j)$.

Thus, we conclude that for a large separation between sites the perturbed resistance approaches the perfect one. To see this, let us consider the removed bond to be between the site $i_o = (0,0,0)$ and the site $j_o = (1,0,0)$, we need to find the resistance between any two sites $i = (i_x, i_y, i_z)$ and $j = (j_x, j_y, j_z)$. To do this one should use Eq. (4.9) and the values given in Table 4. Our results are shown in Table 5. Below we show some examples:

1-The resistance between $i = (0,0,0)$ and $j = (1,0,0)$.

$$R(1,0,0) = R_o(1,0,0) + \frac{\{R_o(1,0,0) + R_o(1,0,0) - R_o(0,0,0) - R_o(0,0,0)\}^2}{4[R - R_o(1,0,0)]}$$

$$= \frac{1}{3}R + \frac{\{\frac{1}{3}R + \frac{1}{3}R - 0 - 0\}^2}{4[R - \frac{1R}{3}]}$$

$$= 0.5R.$$

2-The resistance between $i = (0,0,0)$ and $j = (2,0,0)$

$$R(2,0,0) = R_o(2,0,0) + \frac{\{R_o(1,0,0) + R_o(2,0,0) - R_o(0,0,0) - R_o(1,0,0)\}^2}{4[R - R_o(1,0,0)]}$$

$$= 0.485733R.$$

3- The resistance between $i = (0,0,0)$ and $j = (3,0,0)$

$$R(3,0,0) = R_o(3,0,0) + \frac{\{R_o(1,0,0) + R_o(3,0,0) - R_o(0,0,0) - R_o(2,0,0)\}^2}{4[R - R_o(1,0,0)]}$$

$$= 0.500062R.$$

4- The resistance between $i = (0,0,0)$ and $j = (-4,0,0)$

$$R(-4,0,0) = R_o(4,0,0) + \frac{\{R_o(1,0,0) + R_o(4,0,0) - R_o(0,0,0) - R_o(5,0,0)\}^2}{4[R - R_o(1,0,0)]}$$

$$= 0.5510257R.$$

5- The resistance between $i = (0,0,0)$ and $j = (0,-1,0)$

$$R(0,-1,0) = R_o(0,1,0) + \frac{\{R_o(1,0,0) + R_o(0,1,0) - R_o(0,0,0) - R_o(1,1,0)\}^2}{4[R - R_o(1,0,0)]} = 0.360993R.$$

Table 5: Calculated and measured values of the resistance between the sites $i = (0,0,0)$ and $j = (j_x, j_y, j_z)$, for a perturbed simple cubic lattice (i.e. the bond between $i_o = (0,0,0)$ and $j_o = (1,0,0)$ is broken).

The Site $j = (j_x, j_y, j_z)$	$\frac{R(i,j)}{R}$ Theoretically	$\frac{R(i,j)}{R}$ Experimentally	The Site $j = (j_x, j_y, j_z)$	$\frac{R(i,j)}{R}$ Theoretically	$\frac{R(i,j)}{R}$ Experimentally
(0,0,0)	0	0	**(-1,0,0)**	0.356208	0.3559
(1,0,0)	0.5	0.5009	**(-2,0,0)**	0.454031	0.4565
(2,0,0)	0.485733	0.4904	**(-3,0,0)**	0.4526508	0.5003
(3,0,0)	0.500062	0.5151	**(-4,0,0)**	0.467337	0.5699
(4,0,0)	0.510257	0.5806	**(0,-1,0)**	0.360993	0.3606
(0,1,0)	0.360993	0.3615	**(0,-2,0)**	0.457943	0.4611
(0,2,0)	0.457943	0.4612	**(0,-3,0)**	0.491033	0.5040
(0,3,0)	0.491033	0.5041	**(0,-4,0)**	0.506167	0.5735
(0,4,0)	0.506167	0.5735	**(0,0,-1)**	0.360993	0.3613
(0,0,1)	0.360993	0.3611	**(0,0,-2)**	0.457943	0.4615
(0,0,2)	0.457943	0.4613	**(0,0,-3)**	0.491033	0.5043
(0,0,3)	0.491033	0.5042	**(0,0,-4)**	0.506167	0.5736
(0,0,4)	0.506167	0.5737	**(-1,-1,-1)**	0.454367	0.4560
(1,1,1)	0.4659804	0.4203	**(-2,-2,-2)**	0.50009	0.5170
(2,2,2)	0.503597	0.4780	**(-3,-3,-3)**	0.5158855	0.5854
(3,3,3)	0.517510166	0.5458	**(-4,-4,-4)**	0.5237707	0.8974
(4,4,4)	0.524705	0.8579			

Now, if the removed bond is shifted and becomes between $i_o = (1,0,0)$ and $j_o = (2,0,0)$, then the resistance between any two sites can be calculated, using Eq. (4.9). Our results are arranged in Table 6.

Table 6: Calculated and measured values of the resistance between the sites $i = (0,0,0)$ and $j = (j_x, j_y, j_z)$, for a perturbed SC lattice (i.e. the bond between $i_o = (1,0,0)$ and $j_o = (2,0,0)$ is broken).

The Site $j = (j_x, j_y, j_z)$	$\frac{R(i,j)}{R}$ **Theoretically**	$\frac{R(i,j)}{R}$ **Experimentally**	**The Site** $j = (j_x, j_y, j_z)$	$\frac{R(i,j)}{R}$ **Theoretically**	$\frac{R(i,j)}{R}$ **Experimentally**
(0,0,0)	0	0	**(-1,0,0)**	0.334495	0.3345
(1,0,0)	0.356208	0.3552	**(-2,0,0)**	0.421618	0.4247
(2,0,0)	0.485733	0.4903	**(-3,0,0)**	0.452650	0.4656
(3,0,0)	0.461555	0.4757	**(-4,0,0)**	0.467337	0.5342
(4,0,0)	0.470021	0.5389	**(0,-1,0)**	0.334191	0.3338
(0,1,0)	0.334191	0.3346	**(0,-2,0)**	0.421552	0.4247
(0,2,0)	0.421552	0.4247	**(0,-3,0)**	0.452738	0.4656
(0,3,0)	0.452738	0.4657	**(0,-4,0)**	0.467467	0.5348
(0,4,0)	0.467467	0.5347	**(-1,-1,-1)**	0.420168	0.4185
(1,1,1)	0.419799	0.4218	**(-2,-2,-2)**	0.462590	0.4795
(2,2,2)	0.460461	0.4812	**(-3,-3,-3)**	0.477628	0.5479
(3,3,3)	0.477922	0.5494	**(-4,-4,-4)**	0.485253	0.8602
(4,4,4)	0.485476	0.8616			

4.3 **Experimental results**

To study the resistance of the simple cubic lattice experimentally we constructed a three dimensional SC finite network of identical resistors (R) consisting of ($8x8x8$) resistors, each has a value of ($1\,k\Omega$) and tolerance (1%) as shown in Fig. 2.

Fig. 2 A three dimensional SC mesh consisting of ($8x8x8$) identical resistors.

Using the perfect mesh shown in Fig. 2 above, we measured the resistance between the origin and the site (l,m,n) along the directions [100], [010], [001], and [111]**.** Our results are arranged in Table 7 below.

Table 7: **Calculated and measured values of the resistance between the origin and an arbitrary site in a perfect SC lattice.**

The Site (l,m,n)	$\frac{R_o(l,m,n)}{R}$ Theoretically	$\frac{R_o(l,m,n)}{R}$ Experimentally	The Site (l,m,n)	$\frac{R_o(l,m,n)}{R}$ Theoretically	$\frac{R_o(l,m,n)}{R}$ Experimentally
(0,0,0)	0	0	**(-1,0,0)**	0.3333	0.3333
(1,0,0)	0.3333	0.3331	**(-2,0,0)**	0.419683	0.4230
(2,0,0)	0.419683	0.4227	**(-3,0,0)**	0.450371	0.4635
(3,0,0)	0.450371	0.4633	**(-4,0,0)**	0.464885	0.5321
(4,0,0)	0.464885	0.5323	**(0,-1,0)**	0.3333	0.3337
(0,1,0)	0.3333	0.3331	**(0,-2,0)**	0.419683	0.4228
(0,2,0)	0.419683	0.4228	**(0,-3,0)**	0.450371	0.4634
(0,3,0)	0.450371	0.4623	**(0,-4,0)**	0.464885	0.5322
(0,4,0)	0.464885	0.5321	**(0,0,-1)**	0.3333	0.3335
(0,0,1)	0.3333	0.3334	**(0,0,-2)**	0.419683	0.4231
(0,0,2)	0.419683	0.4230	**(0,0,-3)**	0.450371	0.4635
(0,0,3)	0.450371	0.4634	**(0,0,-4)**	0.464885	0.5324
(0,0,4)	0.464885	0.5325	**(-1,-1,-1)**	0.418305	0.4204
(1,1,1)	0.418305	0.4203	**(-2,-2,-2)**	0.460159	0.4772
(2,2,2)	0.460159	0.4774	**(-3,-3,-3)**	0.475023	0.5464
(3,3,3)	0.475023	0.5461	**(-4,-4,-4)**	0.482570	0.8583
(4,4,4)	0.482570	0.8581			

Now**,** to measure the resistance for the perturbed case we removed the bond between $i_o = (0,0,0)$ and $j_o = (1,0,0)$, then we measured the resistance between the site $i = (0,0,0)$ and the site $j = (j_x, j_y, j_z)$ along the directions [100], [010], [001], and [111]**.** Our results are arranged in Table 5 above.

Now, the removed bond is shifted, $i_o = (1,0,0)$ and $j_o = (2,0,0)$, then we measured again the resistance between the site $i = (0,0,0)$ and the site $j = (j_x, j_y, j_z)$ along the directions [100], [010], [001], and [111]**.** Our results are arranged in Table 6 above.

CHAPTER FIVE

RESULTS AND DISCUSSION

In this chapter, the results of the resistance for two- and three- dimensional infinite networks are given below. Section (5.1) is devoted to the two- dimensional results whereas section (5.2) is concerned with the three- dimensional results.

5.1 Square Lattice

The results of the resistance for the two- dimensional lattice (square) are shown in Figs. (3-12). Figs. (3-6) show the theoretical results for the resistance of the perfect and perturbed infinite square lattices. The resistance diverges as the site (l,m) goes away from the origin.

The figures show the resistance of an infinite square perfect lattice is symmetric under the transformation $(l,m) \to (-l,-m)$**.** This is due to the inversion symmetry of the lattice. However, the resistance of the perturbed infinite square lattice is not symmetric due to the broken bond, except along the [01] direction since the broken bond is along the [10] direction.

Also, one can see that the resistance in the perturbed infinite square lattice is always larger than that in a perfect lattice. This is due to the positive contribution of the second term in Eq. (2.60). But as the separation between the sites increases the perturbed resistance goes to that of a perfect lattice.

Figures (3-4) show that the calculated resistance of the perfect infinite square lattice along the [10] direction is symmetric due to inversion symmetry of the lattice and for large values of l and m, the resistance diverges. While the calculated resistance of the perturbed lattice (the resistor between $i=(0,0)$ and $j=(1,0)$ is broken) is not symmetric and it is always larger than that of the perfect one due to the second positive term in Eq. (2.60).

As the broken bond is shifted to, $i=(1,0)$ and $j=(2,0)$, the calculated perturbed resistance of the lattice approaches that of the perfect one more rapidly. The same thing

can be said about the [01], [12], [21] and [11] directions, except that the calculated perturbed resistance of the lattice is symmetric along the [01] direction, the same behavior as the perfect resistance, because there is no broken bond along this direction, see Figs. (5-6).

The constructed mesh gives accurately the bulk resistance shown in Figs. (7-12), and this means that a crystal consisting of (30x30) atoms enables one to study the bulk properties of the crystal in a good way. But, as we approach the edge then the measured resistance exceeds the calculated one and this is due to the edge effect. Also, one can see from the figures that the measured resistance is symmetric in the perfect mesh, which is expected.

Fig.10 and Fig.12 show that the measured resistance along the [01] direction is nearly symmetric within experimental error, which is expected due to the fact that there is no broken bond along this direction, and this is in agreement with the theoretical result. Finally, our values are in good agreement with the bulk values calculated by Cserti's method.

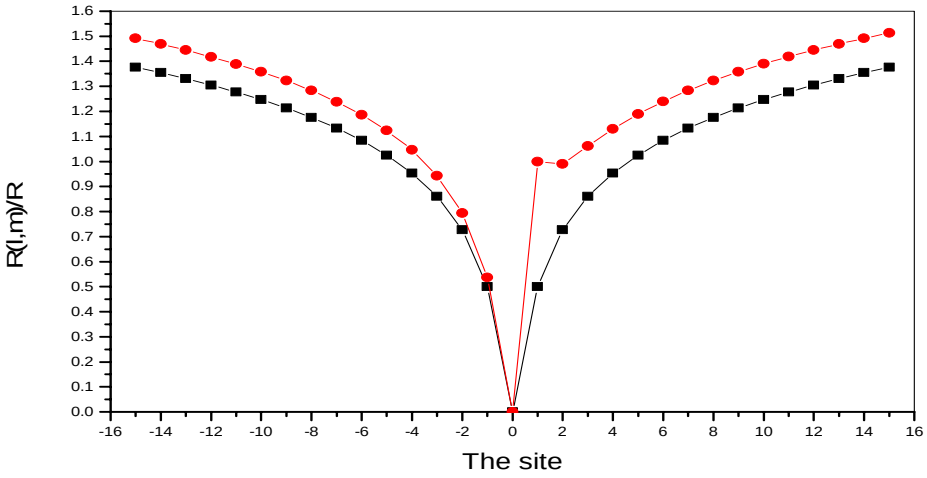

Fig. 3 The calculated resistance between $i=(0,0)$ and $j=(j_x,0)$ along the [10] direction of the perfect (squares) and the perturbed (circles) square lattice as a function of j_x. The ends of the removed bond are $i_o=(0,0)$ and $j_o=(1,0)$.

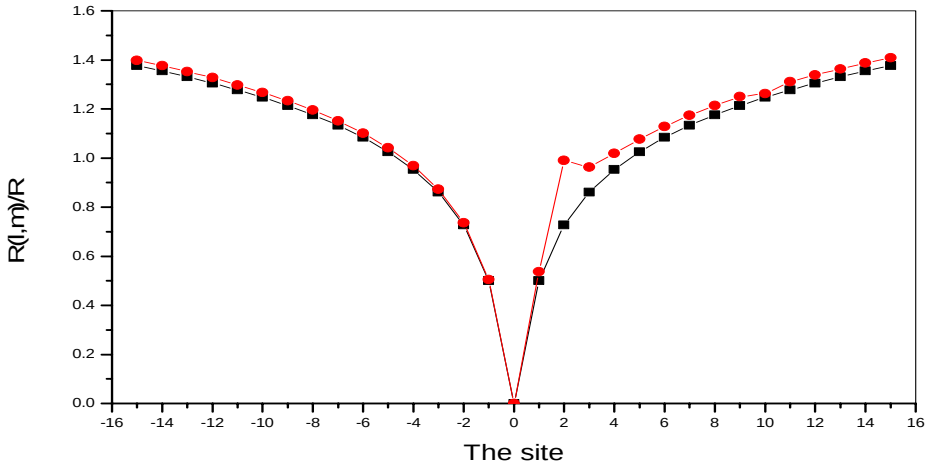

Fig. 4 The calculated resistance between $i=(0,0)$ and $j=(j_x,0)$ along the [10] direction of the perfect (squares) and the perturbed (circles) square lattice as a function of j_x. The ends of the removed bond are $i_o=(1,0)$ and $j_o=(2,0)$.

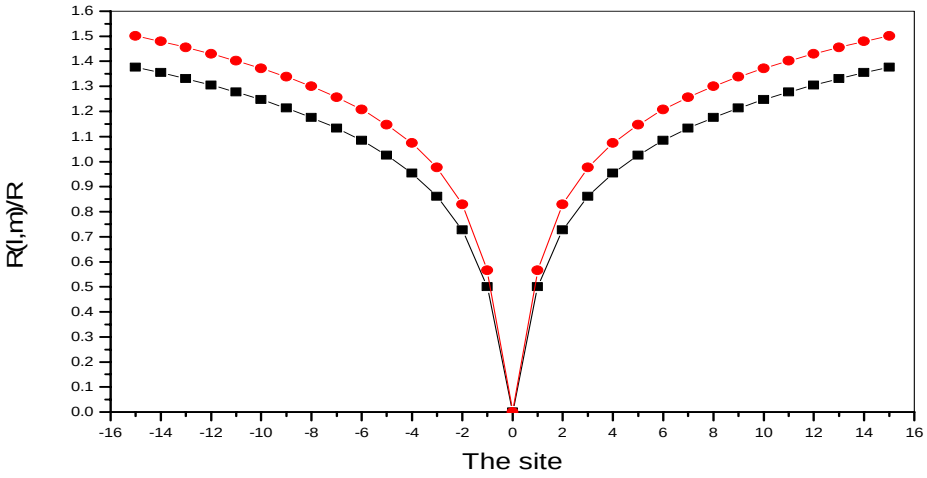

Fig. 5 The calculated resistance between $i=(0,0)$ and $j=(0,j_y)$ along the [01] direction of the perfect (squares) and the perturbed (circles) square lattice as a function of j_y. The ends of the removed bond are $i_o=(0,0)$ and $j_o=(1,0)$.

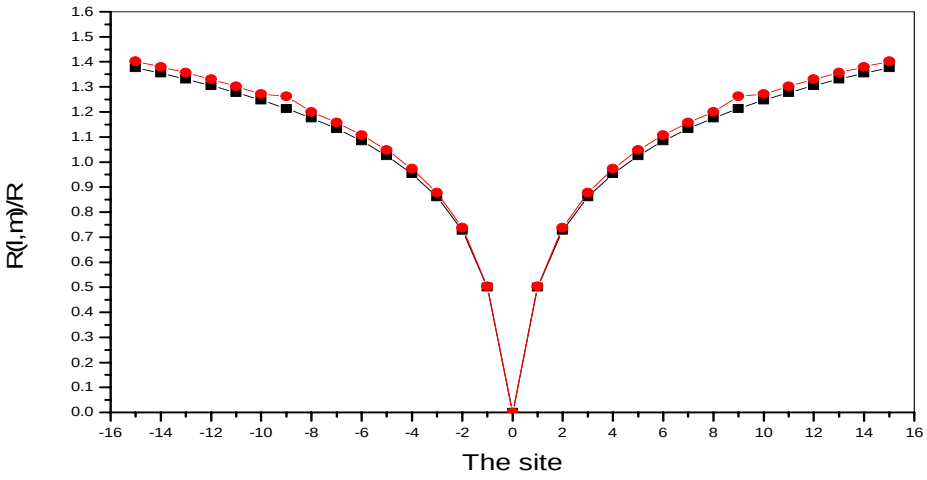

Fig. 6 The calculated resistance between $i=(0,0)$ and $j=(0,j_y)$ along the [01] direction of the perfect (squares) and the perturbed (circles) square lattice as a function of j_y. The ends of the removed bond are $i_o=(1,0)$ and $j_o=(2,0)$.

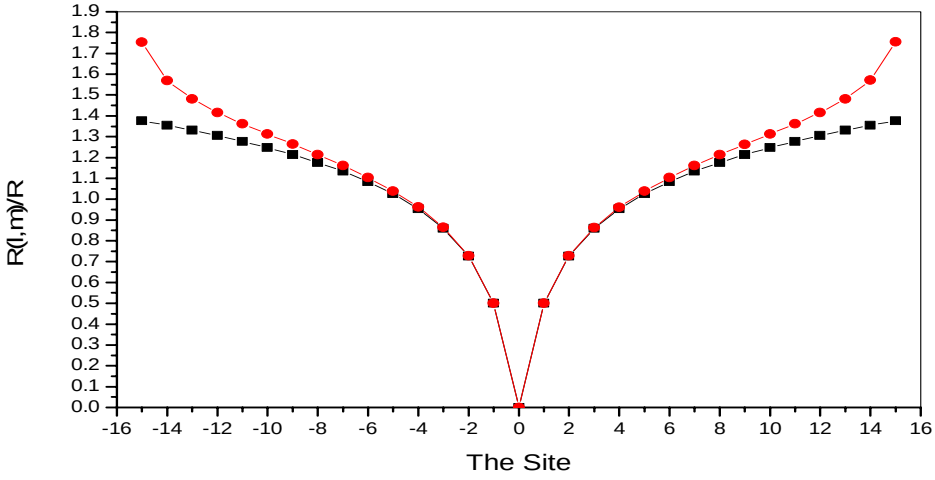

Fig. 7 The resistance between $i = (0,0)$ and $j = (j_x,0)$ of the perfect square lattice as a function of j_x ; calculated (squares) and measured (circles) along the [10] direction.

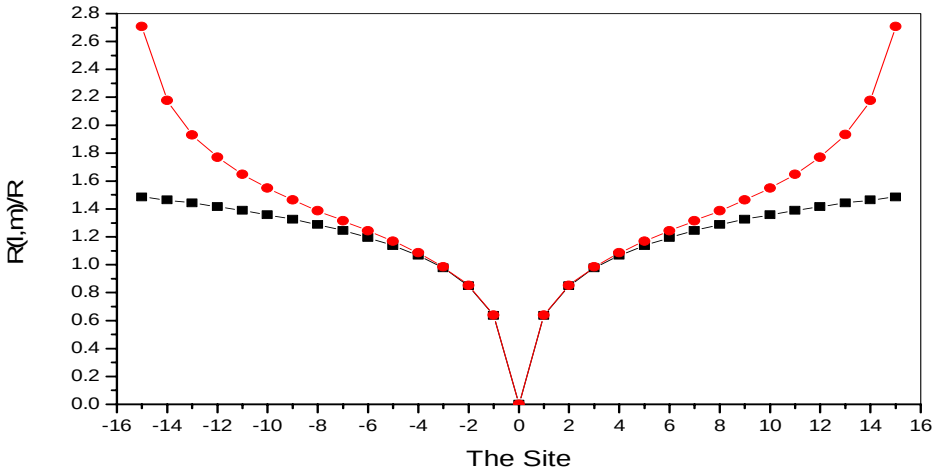

Fig. 8 The resistance between $i = (0,0)$ and $j = (j_x, j_y)$ of the perfect square lattice as a function of j_x and j_y ; calculated (squares) and measured (circles) along the [11] direction.

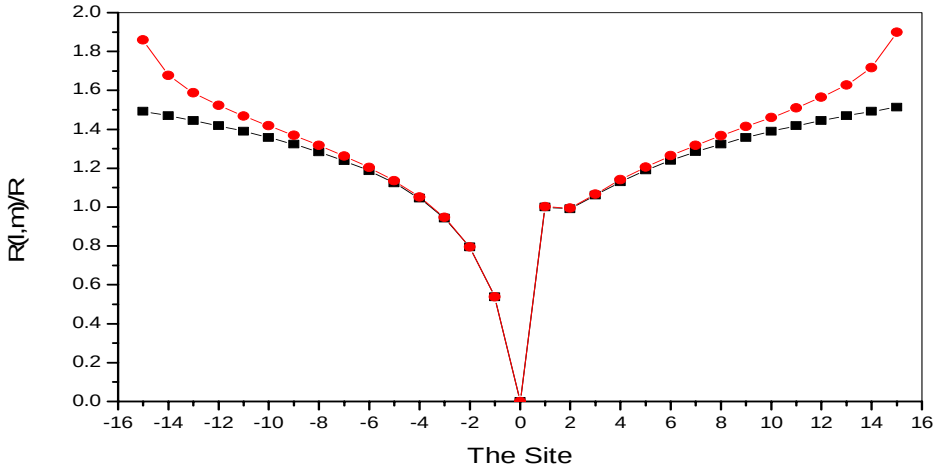

Fig. 9 The resistance between $i = (0,0)$ and $j = (j_x,0)$ of the perturbed square lattice as a function of j_x; calculated (squares) and measured (circles) along the [10] direction. The ends of the removed bond are $i_o = (0,0)$ and $j_o = (1,0)$.

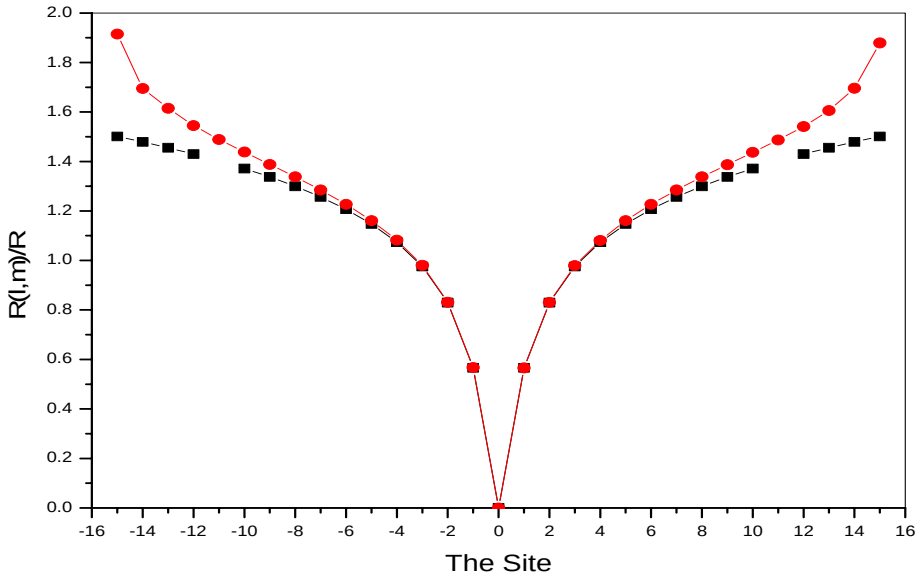

Fig. 10 The resistance between $i = (0,0)$ and $j = (0, j_y)$ of the perturbed square lattice as a function of j_y; calculated (squares) and measured (circles) along the [01] direction. The ends of the removed bond are $i_o = (0,0)$ and $j_o = (1,0)$.

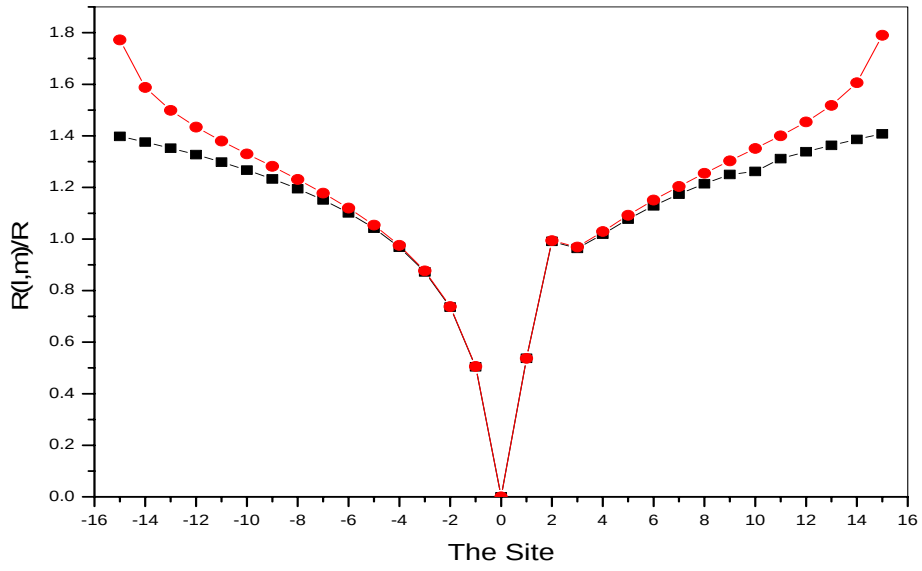

Fig. 11 The resistance between $i = (0,0)$ and $j = (j_x,0)$ of the perturbed square lattice as a function of j_x; calculated (squares) and measured (circles) along the [10] direction. The ends of the removed bond are $i_o = (1,0)$ and $j_o = (2,0)$.

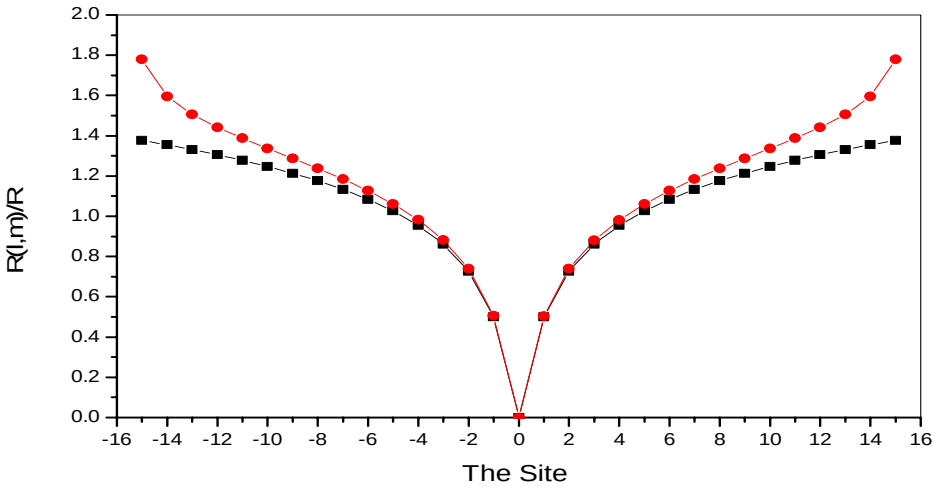

Fig. 12 The resistance between $i = (0,0)$ and $j = (0, j_y)$ of the perturbed square lattice as a function of j_y; calculated (squares) and measured (circles) along the [01] direction. The ends of the removed bond are $i_o = (1,0)$ and $j_o = (2,0)$.

5.2 Simple Cubic (SC) Lattice

The results of the resistance for the three- dimensional lattice (SC) are shown in Figs. (13-26). Figures (13-17) show the theoretical results, and Figs.(18-26) show the experimental ones.

Figure 13 shows the resistance against the site (l,m,n) along the [100] direction for both a perfect infinite and perturbed SC (i.e. the bond between $i_0 = (0,0,0)$ and $j_0 = (1,0,0)$ is broken). It is seen from the figure that the resistance is symmetric (i.e. $R_o(l,0,0) = R_o(-l,0,0)$) for the perfect case due to inversion symmetry of the lattice while for the perturbed case the symmetry is broken, hence the resistance is not symmetric. As (l,m,n) goes away from the origin the resistance approaches its finite value for both cases.

Figure 14 shows the resistance against the site (l,m,n) along the [010] direction for a perfect infinite and perturbed SC (i.e. the bond between $i_0 = (0,0,0)$ and $j_0 = (1,0,0)$ is broken) lattice. The figure shows that the resistance is symmetric for the perfect and perturbed cases, since there is no broken bond along this direction. As (l,m,n) goes away from the origin the resistance approaches its finite value for both cases.

In Figs. (15-17), the same behavior as in the above figures is seen except that the broken bond is shifted (i.e. the bond between $i_0 = (1,0,0)$ and $j_0 = (2,0,0)$ is broken). The resistance along $[100]$ direction is not symmetric in the perturbed case since the broken bond is taken to be along that direction.

From Figs. (13-17), as the broken bond is shifted from the origin along [100] direction then the resistance of the perturbed SC approaches that of the perfect lattice. Also, one can see that the perturbed resistance is always larger than the perfect one.

Figure 18 shows the measured and calculated resistances of the perfect SC lattice against the site (l,m,n) along the [100] direction. It is seen from the figure that the measured resistance is symmetric within the experimental error (i.e. $R_o(l,0,0) = R_o(-l,0,0)$) due to inversion symmetry of the mesh. The measured resistance behaves the same along the directions [010], [001] and [111].

Figure 21 shows the measured and calculated resistance values of the perturbed (i.e. the bond between $i_0 = (0,0,0)$ and $j_0 = (1,0,0)$ is broken) SC lattice against the site (l,m,n) along the [100] direction. It is seen from the figure that the measured resistance is not symmetric (i.e. $R_o(l,0,0) \neq R_o(-l,0,0)$) due to the removed bond. The measured resistance along the [010], [001] and [111] directions is symmetric within experimental errors due to inversion symmetry of the mesh, as shown in Figs. (21-23).

Fig.24 shows the measured and calculated resistance of the perturbed (i.e. the bond between $i_0 = (1,0,0)$ and $j_0 = (2,0,0)$ is broken) SC lattice against the site (l,m,n) along the [100] direction. It is seen from the figure that the measured resistance is not symmetric (i.e. $R_o(l,0,0) \neq R_o(-l,0,0)$) due to the removed bond.

Figures (25-26) show the measured and calculated resistance values of the perturbed (i.e. the bond between $i_0 = (1,0,0)$ and $j_0 = (2,0,0)$ is broken) SC lattice against the site (l,m,n) along the [010] and [111] directions. It is seen from the figures that the measured resistance is symmetric within the experimental error due to the inversion symmetry of the mesh. From Figs. (18-26) the (8x8x8) constructed SC mesh gives the measured bulk resistance nearly exactly as those calculated. This also shows that one can study the bulk properties of a crystal consisting of (8x8x8) atoms accurately. In addition, as we approach the surface of the SC mesh the measured resistance exceeds the calculated due to surface effect.

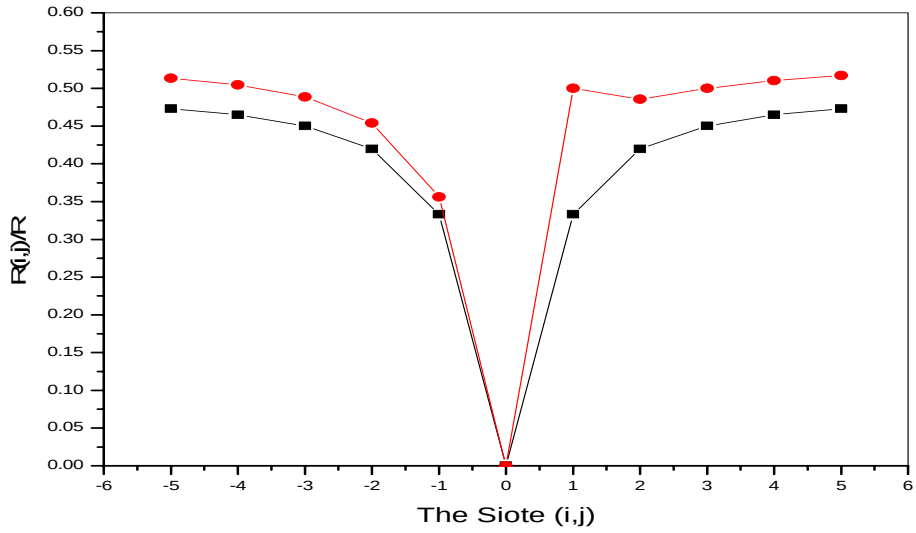

Fig. 13 The resistance on the perfect (squares) and the perturbed (circles) SC between $i = (0,0,0)$ and $j = (j_x,0,0)$ along the [100] direction as a function of j_x. The ends of the removed bond are $i_o = (0,0,0)$ and $j_o = (1,0,0)$.

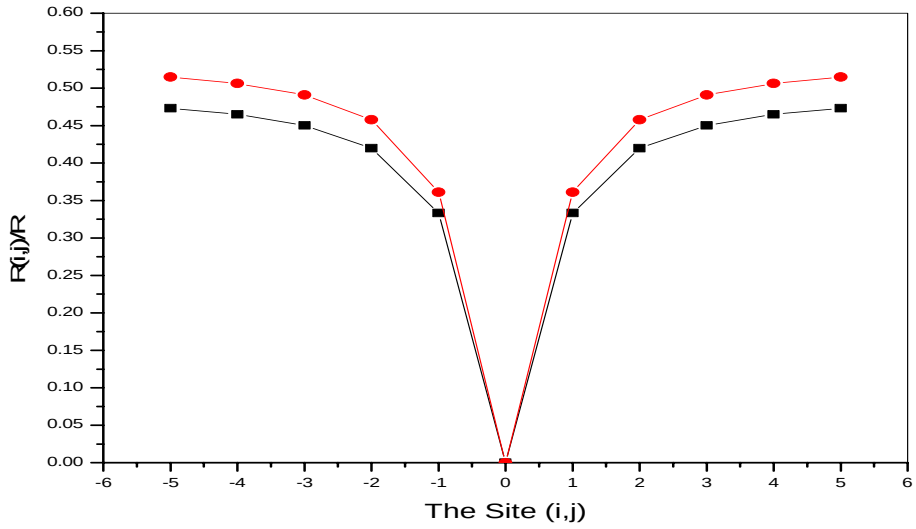

Fig. 14 The resistance on the perfect (squares) and the perturbed (circles) SC between $i = (0,0,0)$ and $j = (0, j_y,0)$ along the [010] direction as a function of j_y. The ends of the removed bond are $i_o = (0,0,0)$ and $j_o = (1,0,0)$.

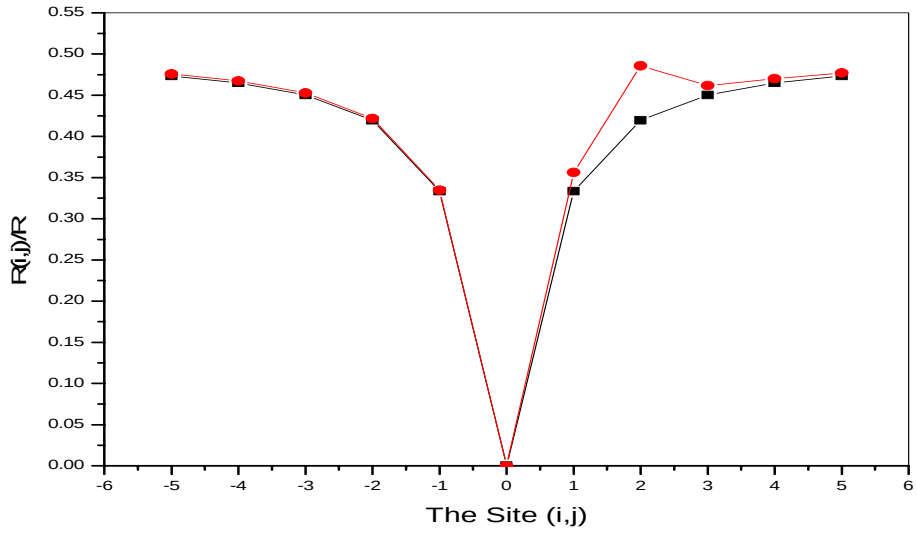

Fig. 15 The resistance on the perfect (squares) and the perturbed (circles) SC between $i = (0,0,0)$ and $j = (j_x,0,0)$ along the [100] direction as a function of j_x. The ends of the removed bond are $i_o = (1,0,0)$ and $i_o = (2,0,0)$.

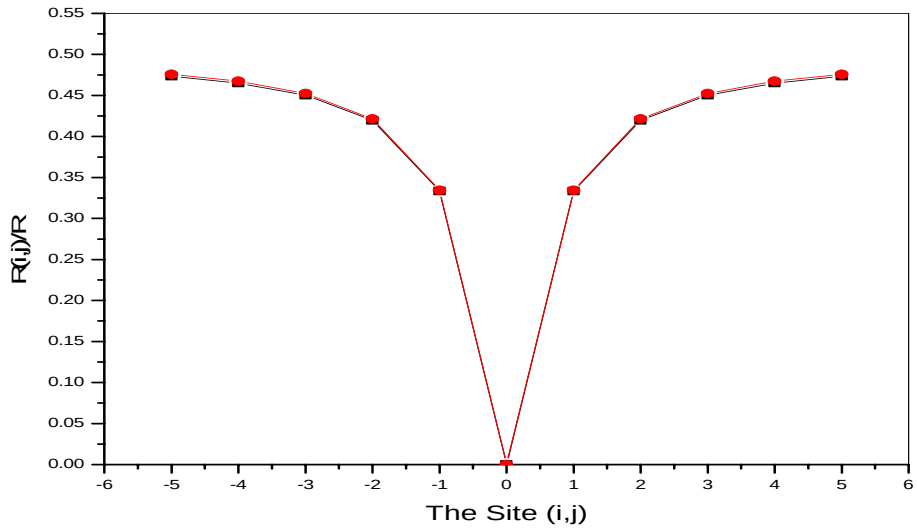

Fig. 16 The resistance on the perfect (squares) and the perturbed (circles) SC between $i = (0,0,0)$ and $j = (0, j_y,0)$ along the [010] direction as a function of j_y. The ends of the removed bond are $i_o = (1,0,0)$ and $i_o = (2,0,0)$.

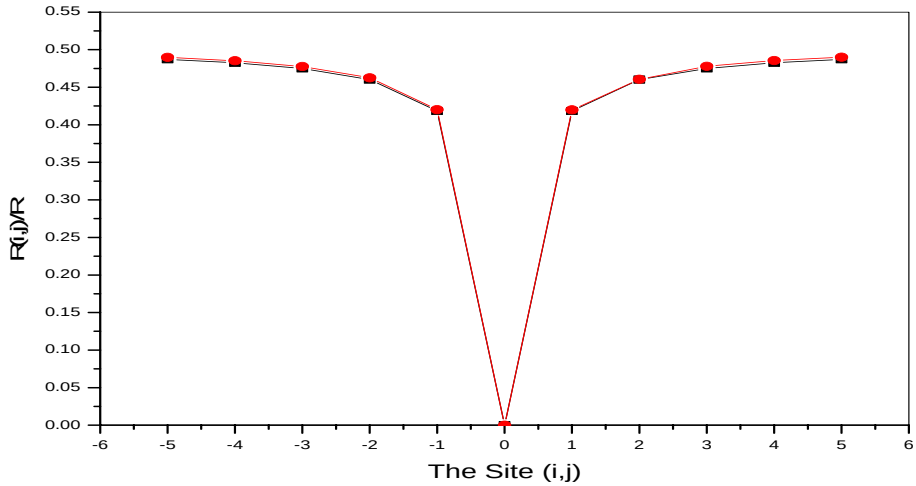

Fig. 17 The resistance on the perfect (squares) and the perturbed (circles) SC between $i = (0,0,0)$ and $j = (j_x, j_y, j_z)$ along the [111] direction as a function of j. The ends of the removed bond are $i_o = (1,0,0)$ and $i_o = (2,0,0)$.

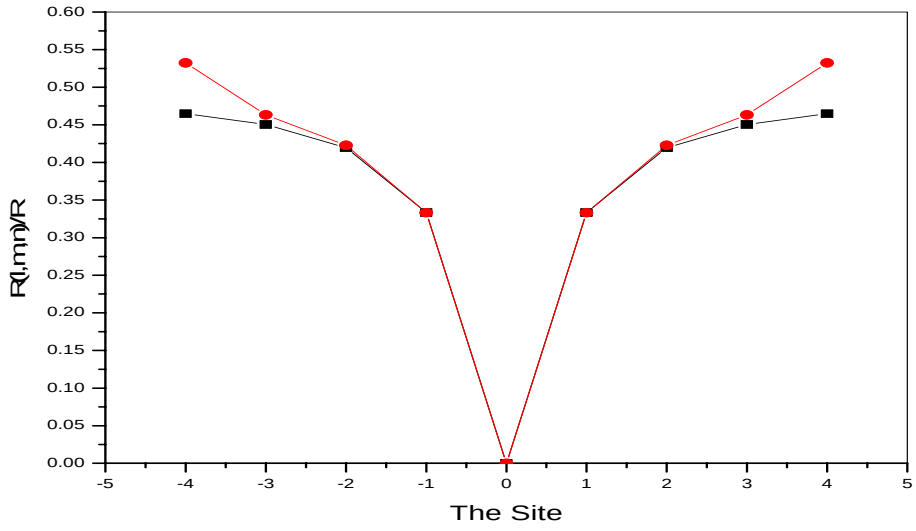

Fig. 18 The resistance between $i = (0,0,0)$ and $j = (j_x,0,0)$ of the perfect SC lattice as a function of j_x; calculated (squares) and measured (circles) along the [100] direction.

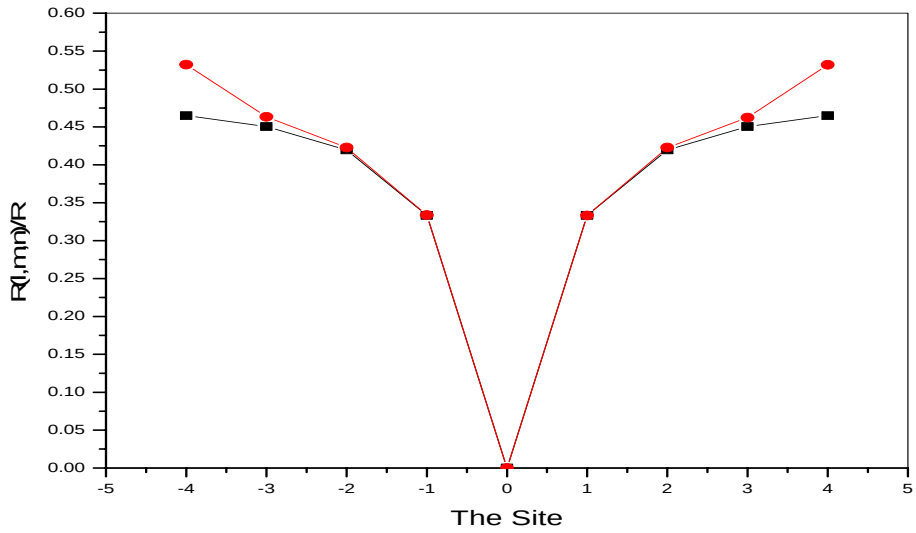

Fig. 19 The resistance between $i = (0,0,0)$ and $j = (0, j_y, 0)$ of the perfect SC lattice as a function of j_y ; calculated (squares) and measured (circles) along the [010] direction.

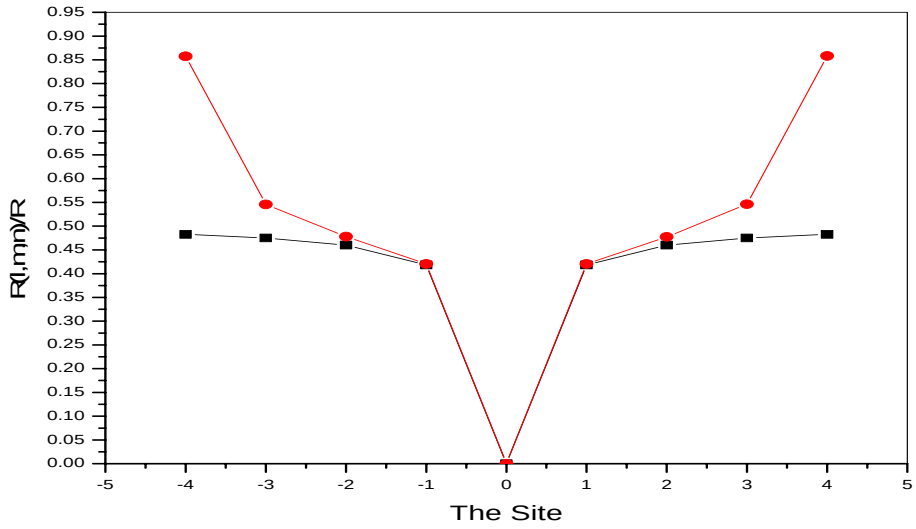

Fig. 20 The resistance between $i = (0,0,0)$ and $j = (j_x, j_y, j_z)$ of the perfect SC lattice as a function of j ; calculated (squares) and measured (circles) along the [111] direction.

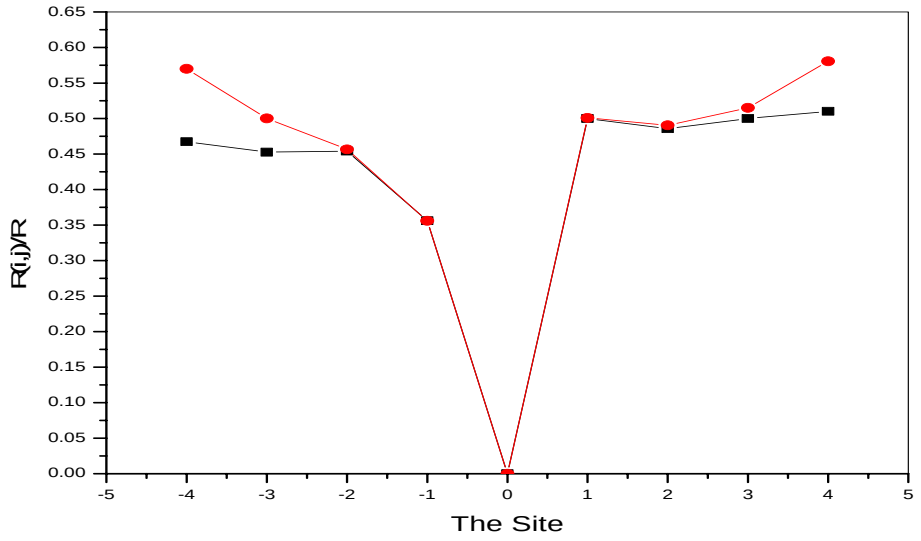

Fig. 21 The resistance between $i = (0,0,0)$ and $j = (j_x,0,0)$ of the perturbed SC as a function of j_x; calculated (squares) and measured (circles) along the [100] direction. The ends of the removed bond are $i_o = (0,0,0)$ and $j_o = (1,0,0)$.

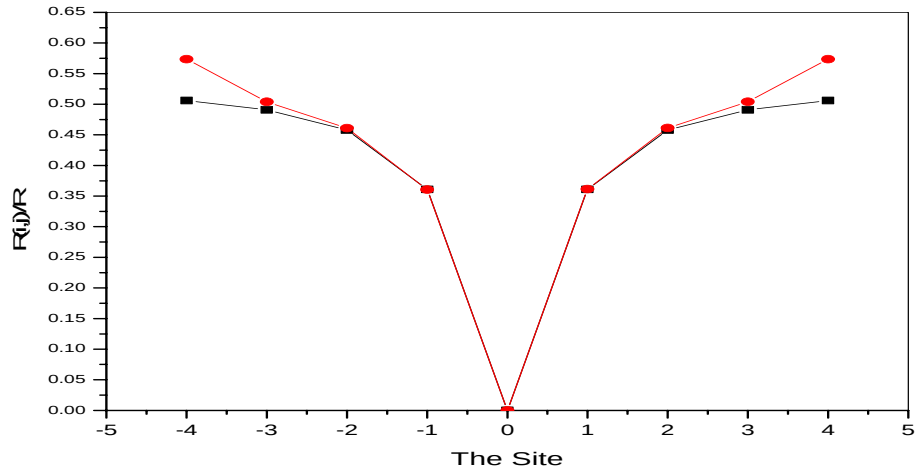

Fig. 22 The resistance between $i = (0,0,0)$ and $j = (0, j_y,0)$ of the perturbed SC as a function of j; calculated (squares) and measured (circles) along the [010] direction. The ends of the removed bond are $i_o = (0,0,0)$ and $j_o = (1,0,0)$.

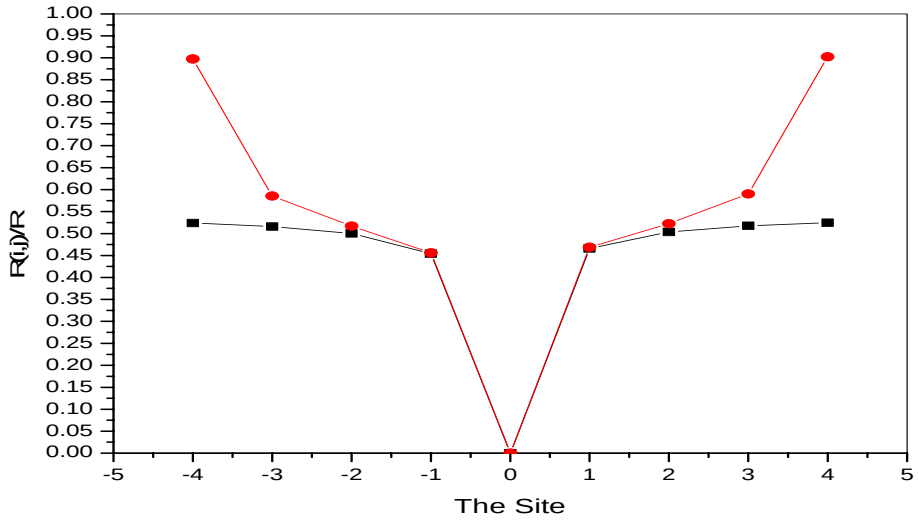

Fig. 23 The resistance between $i = (0,0,0)$ and $j = (j_x, j_y, j_z)$ of the perturbed SC as a function of j; calculated (squares) and measured (circles) along the [111] direction. The ends of the removed bond are $i_o = (0,0,0)$ and $j_o = (1,0,0)$.

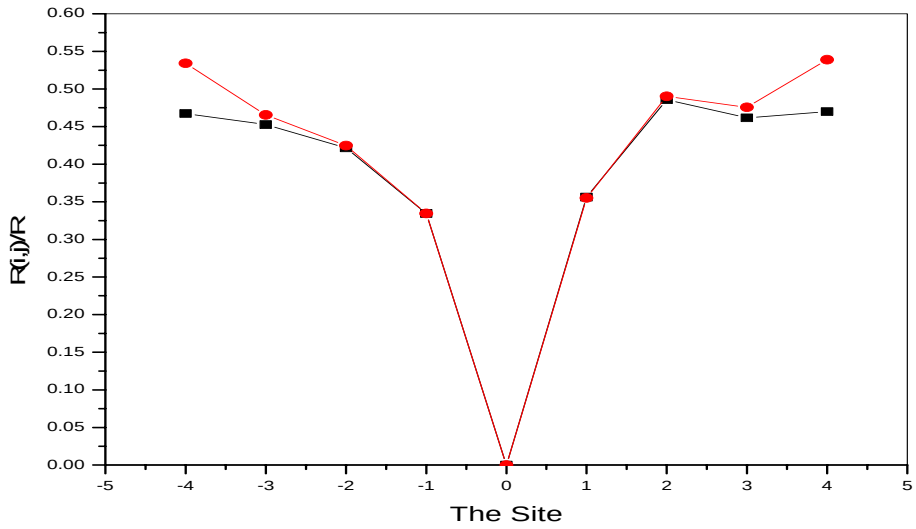

Fig. 24 The resistance between $i = (0,0,0)$ and $j = (j_x,0,0)$ of the perturbed SC as a function of j_x; calculated (squares) and measured (circles) along the [100] direction. The ends of the removed bond are $i_o = (1,0,0)$ and $j_o = (2,0,0)$.

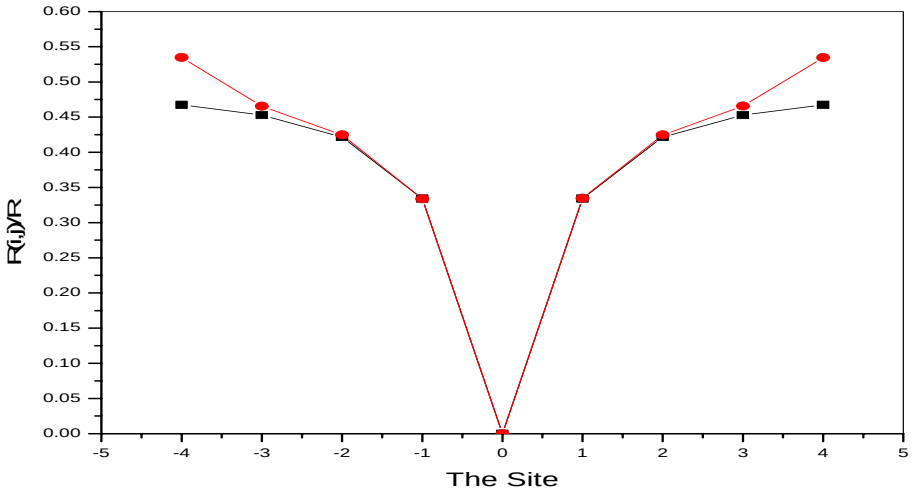

Fig. 25 The resistance between $i = (0,0,0)$ and $j = (j_y,0,0)$ of the perturbed SC as a function of j_y; calculated (squares) and measured (circles) along the [010] direction. The ends of the removed bond are $i_o = (1,0,0)$ and $j_o = (2,0,0)$.

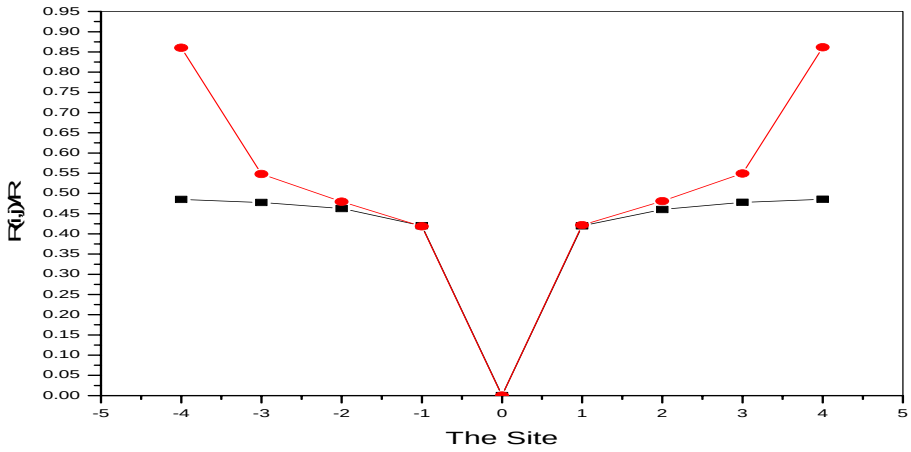

Fig. 26 The resistance between $i = (0,0,0)$ and $j = (j_x, j_y, j_z)$ of the perturbed SC as a function of j; calculated (squares) and measured (circles) along the [111] direction. The ends of the removed bond are $i_o = (1,0,0)$ and $j_o = (2,0,0)$.

CHAPTER SIX

CONCLUSION

In this concluding chapter the highlights of the thesis are first summarized (Section 6.1), and then some open problems and possible extension of this work are presented (Section 6.2).

6.1 General Summary

This work aimed at calculating the resistance between two adjacent points in an infinite network of identical resistors (i.e. square and SC networks) theoretically and experimentally for both the perfect and perturbed cases.

Theoretically, the resistance between adjacent points in a perfect infinite square lattice is written in terms of the LGF at the origin and its derivatives, or by using the so-called recurrence formulae which is simpler. The resistance in a perfect infinite SC lattice is expressed rationally in terms of the LGF at the origin using some recurrence formulae.

Experimentally, the resistance between any two points in a finite square and SC networks is measured for the first time. The bulk values obtained experimentally are very close to those obtained theoretically; while as approaching the edge or the surface of the constructed networks the observed resistances exceed those obtained theoretically due to the edge or the surface effect. It is shown that for large separation between the two sites the resistance in infinite perfect and perturbed square lattice diverges while for infinite perfect and perturbed SC lattices the resistance approaches a finite value.

The theoretical approach used in this thesis may have several advantages:
(i) It can be used for more complicated lattice structures such as body- and face-centered cubic lattices. (ii) The results derived by this method reflect the symmetry of the lattice structures. (iii) From the equation for the Green's function one can, in principle, derive some of the so-called recurrence formulae for the resistances between arbitrary grid points of an infinite lattice. (iv) Finally, our approach for networks of

resistors may serve as a good example for introducing the Green's function method as well as many basic concepts such as the Brillouin zone (BZ) used in solid state physics. We therefore feel that the Green's function method is of some physical interest.

6.2 Open Problems

There are many areas where one can extend the present work:

- One can consider the case where more complicated perturbation is introduced. To do this, one has to write the current contribution at any site $\vec{r}_i$ due to the bonds ($i_o j_o$) and ($k_o l_o$). Write the current at the site $\vec{r}_i$, then removing the above two bonds and writing Ohm's and Kirchhoff's laws as before. Finally, we write Dyson's equation for the new perturbed case and then solve it.

- The problem can be extended to finite lattice and semi infinite lattice structures.

- The Green's function method can be applied to complex systems such as inductivity, capacitance and other combinations.

- One can study finite lattice structures in one dimension, such as ladder structure.

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APPENDICES

APPENDIX A

The Asymptotic Form of the Lattice Green's Functions for a Square Lattice

In this appendix we derive the asymptotic form of the lattice Green's function for a square lattice. The lattice Green's function at site $\vec{r} = 0$ is divergent since $E(\vec{K}) = 0$ for $\vec{K} = 0$. Therefore we calculate the asymptotic form of $G_o(0) - G_o(\vec{r})$. Starting from Eq.(2.21) the LGF for site $\vec{r} = n\vec{a}_1 + m\vec{a}_2$ in a square lattice becomes

$$G_o(0) - G_o(n,m) = \frac{1}{2}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\int_{-\pi}^{\pi}\frac{dx}{2\pi}\frac{1-\exp(inx)\exp(imy)}{2-Cosx-Cosy}. \qquad \text{A1}$$

The integral over x can be obtained from [Cserti, 2000] so,

$$G_o(0) - G_o(n,m) = \frac{1}{2}\int_{-\pi}^{\pi}\frac{dy}{2\pi}\frac{1-\{\exp(-|n|s)\exp(imy)\}}{Sinhs} = \int_{0}^{\pi}\frac{dy}{2\pi}\frac{1-\{\exp(-|n|s)Cosy\}}{\sinh s}. \qquad \text{A2}$$

The same result was obtained by [Venezian, 1994].

A similar method was used in [Chaikin's book, 1995] in the case of a continuous medium in two dimensions. To calculate A2, we break the integral into three parts:

$$G_o(0) - G_o(n,m) = I_1 + I_2 + I_3$$

where

$$I_1 = \int_{0}^{\pi}\frac{dy}{2\pi}\frac{1-\{\exp(-|n|y)Cosmy\}}{y}.$$

$$I_2 = \int_{0}^{\pi}\frac{dy}{2\pi}\left(\frac{1}{Sinhs}-\frac{1}{y}\right).$$

$$I_3 = \int_{0}^{\pi}\frac{dy}{2\pi}\left(\frac{\exp(-|n|y)Cosmy}{y}-\frac{\exp(-|n|s)Cosmy}{Sinhs}\right),$$

where s satisfies $Coshs = 2 - Cosy$. A3

The first part I_1 can be expressed by the integral exponential $Ein(z)$ [Abramowitz et.al, 1972]:

$$I_1 = \frac{1}{2\pi}\mathrm{Re}\{\int_0^{\pi} dy \frac{1-\exp((-|n|-im)y)}{y}. = \frac{1}{2\pi}\mathrm{Re}\{Ein([\pi(|n|-im)]\},$$

where $Ein(z)$ is defined by

$$Ein(z) = \int_0^{z} dt \frac{1-\exp(-t)}{t}.$$

For large values of its argument, $Ein(z) \approx Lnz + \gamma$, where $\gamma = 0.5772...$ is the Euler-Mascheroni constant. Thus, for large n and m I_1 can be approximated by

$$I_1 \approx \frac{1}{2\pi}(Ln|\pi(|n|-im)| + \gamma) = \frac{1}{2\pi}(Ln\sqrt{n^2+m^2} + \gamma + Ln\pi).$$

Using (A3) the integral I_2 can be evaluated exactly:

$$I_2 = \int_0^{\pi} \frac{dy}{2\pi}(\frac{1}{\sqrt{(2-Cosy)^2-1}} - \frac{1}{y}) = \frac{1}{2\pi}(\frac{Ln8}{2} - Ln\pi).$$

In the integral I_3 the integrand is close to zero for small values of y and s since $s \approx Sinhs \approx y$, while for larger values of y and s the exponentials are negligible, therefore $I_3 \approx 0$.

Finally, we find that the LGF for large arguments, i.e., $|\vec{r}| = a\sqrt{n^2+m^2} \to \infty$ becomes

$$G_o(\vec{r}) = G_o(0) - \frac{1}{2\pi}(Ln\frac{|\vec{r}|}{a} + \gamma + \frac{Ln8}{2}). \qquad \text{A4}$$

APPENDIX B

DIVERGENCE OF THE RESISTANCE FOR A PERFECT SQUARE LATTICE

The resistance between the origin and any lattice site (l, m) in a perfect square lattice is given in Eq. (3.29) as (providing that there is a large separation between the origin and the site (l, m))

$$R_o(0;l,m) = \frac{R}{\pi}(Ln\sqrt{l^2+m^2} + \gamma + \frac{Ln8}{2}). \qquad \text{B1}$$

Take the limit of Eq. (B1) as $l \to \infty$. Thus, we can write

$$\lim_{l\to\infty} R_o(l,m) = \frac{R}{\pi}\lim_{l\to\infty}(Ln\sqrt{l^2+m^2} + \gamma + \frac{Ln8}{2});$$

$$= \frac{R}{\pi}\lim_{l\to\infty}(Lnl\sqrt{1+\frac{m^2}{l^2}} + \gamma + \frac{Ln8}{2});$$

$$= \frac{R}{\pi}\lim_{l\to\infty}(Lnl + Ln\sqrt{1+\frac{m^2}{l^2}} + cons\tan t);$$

$$= \frac{R}{\pi}\lim_{l\to\infty}(Lnl + \frac{1}{2}Ln(1+\in) + cons\tan t). \qquad \text{B2}$$

With $\in = \frac{m^2}{l^2} << 1$

The value of $Ln(1+\in)$ can be expanded as [Mary Boas, 1983]

$$Ln(1+\in) = \in - \frac{\in 2}{2} + \frac{\in^3}{3}... \qquad \text{B3}$$

Substituting Eq. (B3) into Eq. (B2) we obtains

$$\lim_{l\to\infty} R_o(l,m) = \frac{R}{\pi}\lim_{l\to\infty}[Lnl + \frac{1}{2}(\in - \frac{\in^2}{2} + \frac{\in^3}{3} + ..) + cons\tan t);$$

$$= \frac{R}{\pi}[Ln\infty + \frac{1}{2}(0) + cons\tan t);$$

$$= \frac{R}{\pi}[\infty + cons\tan t) \to \infty. \qquad \text{B4}$$

So, as $l \to \infty$ then the resistance in a perfect square lattice goes to infinity. The same thing can be said if $m \to \infty$. Therefore, we conclude that as the separation between the origin and the site (l, m) goes to infinity then the resistance diverges to infinity.

APPENDIX C

The Asymptotic form of the Perturbed Resistance for a Square Lattice

Starting with the final expression for the asymptotic behavior of the resistance of the square perfect lattice i.e.

$$R_o(l,m) = \frac{R}{\pi}(Ln\sqrt{l^2+m^2} + \gamma + \frac{Ln8}{2}) \qquad \text{C1}$$

Now, using the above equation we wrote the resistance between the origin and the sites (i, j_o), (j, i_o), (i, i_o), and (j, j_o) respectively.

$$R_o(i, j_o) = \frac{R}{\pi}(Ln\sqrt{i^2 + {j_o}^2} + \gamma + \frac{Ln8}{2});$$

$$R_o(j, i_o) = \frac{R}{\pi}(Ln\sqrt{j^2 + {i_o}^2} + \gamma + \frac{Ln8}{2});$$

$$R_o(i, i_o) = \frac{R}{\pi}(Ln\sqrt{i^2 + {i_o}^2} + \gamma + \frac{Ln8}{2});$$

$$R_o(j, j_o) = \frac{R}{\pi}(Ln\sqrt{j^2 + {j_o}^2} + \gamma + \frac{Ln8}{2}). \qquad \text{C2}$$

By substituting Eq. (C2) into the following formula

$\frac{R(i,j)}{R} = R_o(i,j) + \frac{[R_o(i,j_o) + R_o(j,i_o) - R_o(i,i_o) - R_o(j,j_o)]^2}{4[1 - R_o(i_o, j_o)]}$, One gets the following

$$R(i,j) = R_o(i,j) + \frac{[\frac{R}{\pi}(Ln\sqrt{i^2 + {j_o}^2} + Ln\sqrt{j^2 + {i_o}^2} - Ln\sqrt{i^2 + {i_o}^2} - Ln\sqrt{j^2 + {j_o}^2})]^2}{4[R - R_o(i_o, j_o)]} \qquad \text{C3}$$

Noting that: $Ln(a) + Ln(b) = Ln(ab)$, and $Ln(a) - Ln(b) = Ln(\frac{a}{b})$.

$$R(i,j) = R_o(i,j) + \frac{[\frac{R}{\pi}(Ln\sqrt{i^2 + {j_o}^2}\sqrt{j^2 + {i_o}^2} - Ln\sqrt{i^2 + {i_o}^2}\sqrt{j^2 + {j_o}^2})]^2}{4[R - R_o(i_o, j_o)]}$$

$$R(i,j) = R_o(i,j) + \frac{[\frac{R}{\pi}(Ln\sqrt{\frac{(i^2 + {j_o}^2)(j^2 + {i_o}^2)}{(i^2 + {i_o}^2)(j^2 + {j_o}^2)}}]^2}{4[R - R_o(i_o, j_o)]} \qquad \text{C4}$$

$$R(i,j) = R_o(i,j) + \frac{[\frac{R}{\pi}Ln\sqrt{\frac{i^2 j^2 + i^2 {i_o}^2 + {j_o}^2 j^2 + {j_o}^2 {i_o}^2}{i^2 j^2 + i^2 {j_o}^2 + {i_o}^2 j^2 + {i_o}^2 {j_o}^2}}]^2}{4[R - R_o(i_o, j_o)]} \qquad \text{C5}$$

Now taking the limit of the Eq. (C5) as both i and j goes to infinity and using L' Hopitals rule, then the second term cancels out $\Rightarrow$

$$\underset{i,j\to\infty}{Lim} R(i,j) \to R_o(i,j) \to \infty \qquad \text{C6}$$

APPENDIX D

ASYMPTOTIC FORM OF THE RESISTANCE FOR A SC LATTICE

The resistance between the origin and any lattice site (l, m, n) in a perfect SC lattice is given in Eq. (3.27) as:

$$\frac{R_o(l,m,n)}{R} = [G_o(0,0,0) - G_o(l,m,n)] \qquad \text{D1}$$

Now, the LGF for a perfect SC lattice is given as [Economou, 1983]

$$G_o(l,m,n) = (\frac{1}{\pi^3}) \int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \frac{\cos lx \cos my \cos nz}{E - \cos x - \cos y - \cos z} dxdydz \qquad \text{D2}$$

Taking the limit of Eq. (D2) as $l \to \infty$, then we may write

$$\lim_{l\to\infty} G_o(l,m,n) = (\frac{1}{\pi^3}) \lim_{l\to\infty} \int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \frac{\cos lx \cos my \cos nz}{E - (\cos x + \cos y + \cos z)} dxdydz \qquad \text{D3}$$

$$= (\frac{1}{\pi^3}) \int_0^{\pi}\int_0^{\pi} [\lim_{l\to\infty} \int_0^{\pi} \frac{\cos lx}{E - (\cos x + \cos y + \cos z)} dx] \cos my \cos nz dydz \qquad \text{D4}$$

Now, let us take I to be

$$I = \lim_{l\to\infty} \int_0^{\pi} \frac{\cos lx}{E - (cox + \cos y + \cos z)} dx;$$

$$= \lim_{l\to\infty} \int_0^{\pi} \phi(x) \cos lx dx. \qquad \text{D5}$$

In the theory of Fourier series, we have the so-called Riemann's lemma i.e.:

$$\lim_{p\to\infty} \int_a^b \phi(x) \cos px dx \to 0. \qquad \text{D6}$$

From Eq. (D6), we conclude that $I = 0$. Thus, Eq. (D4) becomes

$$\lim_{l\to\infty} G_o(l,m,n) \to 0. \qquad \text{D7}$$

The same thing can be done for $m \to \infty$ and for $n \to \infty$. Thus, we conclude that the LGF for a perfect SC lattice goes to zero as any of l, or m, or n goes to infinity.

Finally, Eq. (D1) becomes

$$\frac{R_o(l,m,n)}{R} \to G_o(0,0,0). \qquad \text{D8}$$

So the resistance in a perfect SC lattice goes to a finite value for large separation between the origin and the site (l,m,n).

ملخـــص

حسـاب مقـاومـة شـبكيـات لا نهائية-تطـبيـق علـى دالـــة غريـن

إعـداد الطـــالب
جهـــــــــــــــاد أســـــــــــــعد
إشـــــراف
أ. د. جميـــــــــــل خليفـــــــــــة

لقـد تـم حسـاب المقـاومـة الكهـربـائـية لشبكيات لا نهائية في بعدين وفي ثلاثة أبعاد، وذلك باستخدام دالة غرين. وتناول البحث حالتين هما: الحالة المثالية للشبكيات اللانهائية (بعدين وثلاثة أبعاد)، والحالة اللامثالية(المشابة) للشبكيات، حيث تم إزالة رابطة بين نقطتين شبكيتين.

وجرى توضيح كيفية اشتقاق العلاقة الرئيسة التي تربط المقاومة الكهربائية لشبكية لانهائية باستخدام دالة غرين لكلا الحالتين السابقتين (المثالية واللامثالية). أما بالنسبة لحساب المقاومة الكهربائية للشبكية الثنائية فقد تم استعمال طريقتين: الأولى باستخدام قيم دالة غرين عند نقاط شبكية محددة، والثانية باستخدام بعض علاقات التكرار.

وتم التعبير عن المقاومة الكهربائية لشبكيات لا نهائية ثلاثية الابعاد بشكل نسبي وبدلالة دالة غرين عند نقطة الاصل فقط . كما تم التأكد من الحساب النظري ببناء شبكية ثنائية الأبعاد تتكون من (20x20) مقاومة متماثلة وأخرى ثلاثية الأبعاد تتكون من (8x8x8) مقاومة متماثلة، وقد بينت النتائج العملية تقارباً ممتازاً مع النموذج النظري.