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Simple shearing and azimuthal shearing of an internally balanced compressible elastic material

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ABSTRACT

The finite deformation response of a compressible internally balanced elastic material is studied for deformations that involve progressive shearing. The internally balanced material theory requires that an equation of internal balance is satisfied at each material point. This arises from the constitutive theory which makes use of a multiplicative decomposition of the deformation gradient. Satisfaction of the internal balance requirement then yields the most energetically favorable decomposition. Here we consider a particular compressible internally balanced material model that is motivated by a Blatz–Ko type energy from the conventional hyperelastic theory. The conventional hyperelastic theory occurs as a special limiting case of the internally balanced constitutive theory. More generally, the internally balanced material exhibits softer mechanical behavior. This gives rise to a stress-plateau in the simple shearing response whereas such plateaus do not occur in the corresponding hyperelastic treatment. The boundary value problem for azimuthal shearing with a possible radial stretching is then studied. The internally balanced material response is again found to be softer than that of the hyperelastic limiting case. This is manifest in terms of an upper bound to the applied twisting moment for the existence of solutions to the boundary value problem. In contrast, the hyperelastic limiting case has solutions for all values of applied moment.

1. Introduction

In finite deformation solid mechanics the deformation gradient $\mathbf{F}=\partial\mathbf{x}/\partial\mathbf{X}$ is central to the kinematic description. Here $\mathbf{x}=\chi(\mathbf{X})$ is the mapping from the reference location $\mathbf{X}$ to the current location $\mathbf{x}$. The theory of hyperelasticity makes use of $\mathbf{F}$ to develop its constitutive theory in terms of the elastic stored energy density $W=W(\mathbf{C})$ where $\mathbf{C}=\mathbf{F}^{\mathrm{T}}\mathbf{F}$. In the absence of internal material constraints, the Cauchy stress $\mathbf{T}$ is then given by

$$\mathbf{T}=\frac{2}{J}\mathbf{F}\frac{\partial W}{\partial\mathbf{C}}\mathbf{F}^{\mathrm{T}} \tag{1}$$

with $J=\det\mathbf{F}$. The stress equations of equilibrium in the absence of body forces take the well known form

$$\operatorname{div}\mathbf{T}=\mathbf{0} \tag{2}$$

where div is the divergence operator with respect to current configuration $\mathbf{x}$.

More general treatments of solid material behavior, specifically those that seek to describe how a combination of elastic and inelastic effects govern large deformation, often make use of a multiplicative decomposition of $\mathbf{F}$, say

$$\mathbf{F}=\hat{\mathbf{F}}\mathbf{F}^{*}. \tag{3}$$

This includes the Kröner–Lee multiplicative decomposition for the treatment of finite deformation plasticity [1,2], as well as descriptions of growth and remodeling in biological tissue (e.g., [3,4]). The standard modeling scenario when invoking (3) involves $\mathbf{F}^{*}$ describing the inelastic part of the process after which $\hat{\mathbf{F}}$ provides some elastic accommodation. The scientific literature in this area is now vast, and new types of physical phenomena are regularly being described using such a decomposition [5–7]. Because elastic and inelastic effects may permeate all aspects of a complex physical process, decomposition sequences in which elastic and inelastic factors alternate with each other can also logically be considered (e.g., [8]). This motivates the consideration of (3) in a context where both $\hat{\mathbf{F}}$ and $\mathbf{F}^{*}$ are each associated with a separate purely elastic type of effect. A theory of internally balanced elastic materials emerges under such considerations. Because the conventional theory of hyperelasticity (meaning the theory which does not invoke (3)) provides useful simplifications under the

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