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A constitutive model for an internally balanced compressible elastic material

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Abstract
Many large deformation constitutive models for the mechanical behavior of solid materials make use of the multiplicative decomposition $\mathbf{F} = \hat{\mathbf{F}}\,\mathbf{F}^*$ as, for example, used by Kröner in the context of finite-strain plasticity. Then $\hat{\mathbf{F}}$ describes the elastic effect by letting the potential energy of the deformation depend upon $\hat{\mathbf{F}}$. In this paper we allow the potential energy to depend upon both portions of the multiplicative decomposition. As in hyperelasticity, energy minimization with respect to displacement gives equilibrium field equations and traction boundary conditions. The new feature, *minimization with respect to the decomposition itself*, generates an additional mathematical requirement that is interpreted here in terms of a principle of internal mechanical balance. We specifically consider a Blatz–Ko-type solid suitably generalized to incorporate the notion of internal balance. Conventional results of hyperelasticity are retrieved for certain limiting forms of the energy density, whereas the general form of the energy density gives rise to an overall softening response, as is demonstrated in the context of pure pressure and uniaxial loading.

Keywords
Hyperelasticity, internal balance, stress softening, substructure modeling, energy minimization

1. Introduction

The theory of hyperelasticity develops its constitutive framework in terms of a stored energy density $W = W(\mathbf{F})$, where $\mathbf{F} = \nabla\boldsymbol{\chi}$ is the deformation gradient tensor for a deformation that describes deformed locations $\mathbf{x}$ in terms of reference locations $\mathbf{X}$ by means of the mapping $\mathbf{x} = \boldsymbol{\chi}(\mathbf{X})$. The gradient operation ∇ is then with respect to the reference coordinate locations, and the energy density W is then per unit volume in this reference configuration. More generally, in finite-deformation continuum mechanics, the multiplicative decomposition of the deformation gradient

$$\mathbf{F} = \hat{\mathbf{F}}\,\mathbf{F}^* \tag{1}$$

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