FE Formulation of Internal Mechanical Balance based on Multiplicative Decomposition of the Deformation Gradient

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The multiplicative decomposition of the deformation gradient $\mathbf{F}=\hat{\mathbf{F}}\,\mathbf{F}^*$ is widely used in the mechanical description of large deformation material behavior of solids. Then, typically, only the $\hat{\mathbf{F}}$ portion of the decomposition is regarded as hyperelastic. Recent theoretical studies have explored the implications of material modeling if both $\hat{\mathbf{F}}$ and $\mathbf{F}^*$ are regarded as hyperelastic. The resulting theory holds promise for describing a variety of effects because of an internal balance principle that emerges naturally from the overall variational treatment. However in order for such models to enjoy practical usage, it is necessary to provide a finite element framework. The present work is a first step in that direction. We present the weak form for compressible materials in a Total Lagrange framework. The FE formulation is implemented via in–house code and demonstrated in the context of uniaxial loading of one tetrahedral element. The achieved stress by the FE formulation is in a good agreement with the analytically calculated stress.

Keywords: *Finite Element, Total Lagrange, Internal Balance, Multiplicative Decomposition*

1 Introduction

Nonlinear continuum mechanics is commonly used to study the response of highly deformable materials. Nowadays, the proposed material models often include internal material variables in order to capture realistic engineering behavior. The theoretical background is mature and its implementation in the frame of finite element method is well established. Typically, these material constitutive models are expressed in terms of the deformation gradient $\mathbf{F}$, its derived quantities, and various multiplicative decompositions, e.g.,

$$\mathbf{F} = \hat{\mathbf{F}}\,\mathbf{F}^*. \tag{1}$$

The multiplicative decomposition Eq. (1) typically serves to assign particular portions of $\mathbf{F}$ to specific parts of the material response. The usual scheme is that $\hat{\mathbf{F}}$ models elastic response and it is associated to the rules of variational calculus. The $\mathbf{F}^*$ portion then models inelastic response, often by means of a time dependent evolution law. All of the tensors $\mathbf{F}$, $\hat{\mathbf{F}}$ and $\mathbf{F}^*$ are invertible and taken to be functions of location $\mathbf{X}$ in the reference configuration. Although $\mathbf{F}$ is the gradient of the deformation map from reference locations $\mathbf{X}$ to deformed locations $\mathbf{x}$, the tensors $\hat{\mathbf{F}}$ and $\mathbf{F}^*$ in Eq. (1) need not themselves be gradients of mappings. In other words, when using Eq. (1) it is not at all necessary for there to be a specific local intermediate configuration such that $\mathbf{F}^*$ associates with a mapping to such an intermediate configuration and $\hat{\mathbf{F}}$ associates with a mapping from such an intermediate configuration.

Recent theoretical studies have explored the consequences of material modeling if both $\hat{\mathbf{F}}$ and $\mathbf{F}^*$ are regarded as hyperelastic [1]. The implications, demonstrated in the context of an incompressible type theory, show among other things the possibility of modeling slip zones and other phenomena that typically require ad hoc assumptions in more standard treatments. Such ad hoc assumptions are not necessary in [1] because of an internal balance principle that emerges naturally from the resulting variational treatment. This in turn has motivated the theoretical consideration of compressible versions of such a framework [2]. However in order for such models to enjoy practical usage, it is necessary to provide a finite element framework. In this work we demonstrate how the FE formulation of this internal balance multiplicative decomposition is naturally achieved by linearizing the weak form that is obtained by the variation with respect to both portions of the multiplicative decomposition. The derivation is expressed initially in the frame of Total Lagrange Formulation. Then, uniaxial loading of one tetrahedral element is presented.

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