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Modeling of Soft Materials via Multiplicative Decomposition of Deformation Gradient

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Finite elasticity is widely used to investigate the response of highly deformable engineering materials. In general for large deformations, the deformation gradient $\mathbf{F}$, and it's derived quantities, is the basic asset to express stored energy of hyperelastic material model. If other mechanical effects are present in addition to finite elasticity then the decomposition of $\mathbf{F} = \hat{\mathbf{F}}\mathbf{F}^*$ is commonly used. This serves to distinguish and relate particular portions of $\mathbf{F}$ to specific parts of the material response. The usual scheme is then that $\hat{\mathbf{F}}$ models elastic response and it is associated to the rules of variational calculus. The $\mathbf{F}^*$ portion then models inelastic response, usually by means of a time dependent evolution law. Recently, the arguments of variational calculus have been applied to both portions of the deformation gradient decomposition for hyperelastic material [1, 2]. The decomposition itself is then determined by an additional internal balance equation that is generated by such a variational treatment. To demonstrate let us recall a special case of the generalized Blatz–Ko material model from hyperelasticity, see [3], that has a stored energy function $W(\mathbf{F})$ in the form

$$W(I_2, I_3) = \mu\,(I_2/I_3 + 2I_3^{1/2} - 5)/2, \tag{1}$$

where $\mu > 0$ is a material parameter that can be interpreted as a shear modulus. The variables I_2 and I_3 denote the second and the third principal scalar invariants of $\mathbf{C} = \mathbf{F}^{\mathrm{T}}\mathbf{F}$. In analogy to (1) the internally balanced material treatment, we consider a stored energy function $W(\mathbf{F}, \hat{\mathbf{F}})$ in the form

$$W(\hat{I}_2, \hat{I}_3, I_2^*, I_3^*) = \hat{\mu}\,(\hat{I}_2/\hat{I}_3 + 2\hat{I}_3^{1/2} - 5)/2 + \mu^*\,(I_2^*/I_3^* + 2I_3^{*\,1/2} - 5)/2, \tag{2}$$

where $\hat{\mu} > 0$ and $\mu^* > 0$ are material parameters that generalize the role of μ in (1). The variables $\hat{I}_2$ and $\hat{I}_3$ are the corresponding principal scalar invariants of $\hat{\mathbf{C}} = \hat{\mathbf{F}}^{\mathrm{T}}\hat{\mathbf{F}}$. The variables I_2^* and I_3^* are the corresponding principal scalar invariants of $\mathbf{C}^* = \mathbf{F}^{*\mathrm{T}}\mathbf{F}^*$.

Uniaxial Loading Consider a cube of such material with unit length sides in the reference configuration. The principal stretching λ_1 then stores a total energy $w(\lambda_1, \lambda_2) \times 1$ where λ_2 is lateral contraction. The stored energy function $w(\lambda_1, \lambda_2)$ expresses a reduced form of (1) as

$$w(\lambda_1, \lambda_2) = \mu\,(2\lambda_2^{-2} + \lambda_1^{-2} + 2\lambda_1\lambda_2^2 - 5)/2. \tag{3}$$

Let P be the applied longitudinal force that is uniformly distributed and it follows then $P = T_{\mathrm{xx}}\,\lambda_2^2$ where T_{xx} is longitudinal Cauchy stress component. Equilibrium under the applied load requires minimization of the $\mathscr{E}(\lambda_1, \lambda_2) = w(\lambda_1, \lambda_2) - P(\lambda_1 - 1)$ with respect to λ_1 and λ_2 and that gives

$$\lambda_2 = \lambda_1^{-1/4}, \qquad T_{\mathrm{xx}} = \mu\,(1 - \lambda_1^{-5/2}). \tag{4}$$

Now consider the internally balanced material with W given by (2). Then $\mathbf{F}$ is decomposed as

$$\lambda_1 = \hat{\lambda}_1 * \lambda_1^*, \qquad \lambda_2 = \hat{\lambda}_2 * \lambda_2^*, \tag{5}$$