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Electrical characteristics of the 2 × *n* and ☐ × *n* circuit network

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Abstract

This paper presents two new fundamentals of the $2 \times n$ and $\square \times n$ circuit network. The results of a plane $2 \times n$ resistor network can be applied to a $\square \times n$ circuit network, which has not been studied before. We first study the $2 \times n$ resistor network by modeling a differential equation and obtain two equivalent resistances between two arbitrary nodes of the $2 \times n$ network. Next, the $\square \times n$ cube network is transformed to the $2 \times n$ plane network equivalently to achieve two resistance formulae between two arbitrary nodes of the $\square \times n$ cube network. By applying the resistance results to the $\square \times n$ *LC* cube network, the complex impedance characteristics of the *LC* network, which includes oscillation characteristics and resonance properties, are discovered.

Keywords: $\square \times n$ circuit network, equivalent transform, differential equation, resonance properties

1. Introduction

Resistor networks cover many fields of study for both physicists and electrical engineers thanks to Kirchhoff (a German physicist) who formulated the basic laws of electric circuits in 1845 (i.e. the current law and the voltage law). Since then, many resistor network models have been investigated in previous research [1–10]. Modern science has made great progress thanks to the development and application of circuit theory. Therefore, all kinds of problems in the circuit field are worth being researched and explored; the equivalent resistance of resistor networks is of such interest and deserves to be studied for a variety of subjects [11–46]. On the other hand, the resistor network model can be used to explore and solve many complex problems for both electrical and non-electrical problems [12–21].

Infinite networks consisting of either identical resistors or identical capacitors have been the subject of much research effort for a long time. Three methods and techniques that have been developed are mainstream in investigating the infinite resistor networks, including the current distribution method [1], the lattice Green's function (LGF) method [3–10] and the random walk method [11, 12]. The LGF method is important because it enables us to study infinite perfect networks in addition to perturbed infinite networks, as one can see in the previous works carried out.

In many cases, most networks that need to be studied and considered are finite. In 2004, Wu [22] calculated the equivalent resistance between two arbitrary nodes in resistor networks, and his method is based on the Laplacian matrix approach. Following the Laplacian matrix method formulated by Wu, many modified and developed achievements have been obtained [23–26].

Later on, a new recursion-transform (RT) method was established by Tan [27] in 2011. The RT method enables us to calculate the equivalent resistance of a finite $m \times n$ resistor network with different boundaries, including the cobweb model [28–31], hammock network [26], globe network [32], fan network [33], and rectangular network [34, 35]. In addition, Tan improved his RT method which can be applied to many resistor networks with arbitrary boundaries [36–40] and the complex impedance network [34, 35, 40]. Now, the RT method has been extended from calculating resistance to researching the node potential [41–44].

As a summary, the RT method can be expressed by three forms (RT-I, RT-V, N-RT). The first form is the RT-I method,

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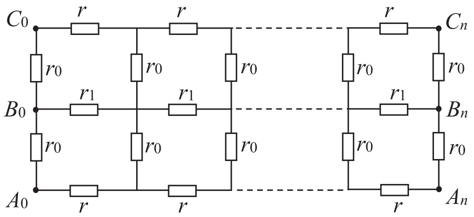

Figure 1. Geometry of the $2 \times n$ plane network with three resistor elements of r_0, r and r_1.

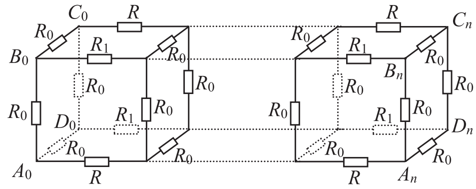

Figure 2. Geometry of the n-step cube network with three resistor elements of R_0, R and R_1.

which uses current parameters to build a matrix equation and the branch currents and equivalent resistance can be easily evaluated [35–40]. The second form is the RT-V method, which uses potential parameters to build a matrix equation and the node potential and equivalent resistance can be easily evaluated [41–44]. The third form is the N-RT method, which uses resistance parameters to build a fractional difference equation to study the complex n-step network, and the equivalent resistance of the complex n-step resistor network can be easily evaluated [45, 46]. These three different methods can be used respectively according to different conditions.

With the development of natural science, many new problems with the resistor network constantly arise. The calculation of the equivalent resistance is difficult when dealing with complex network models, and the expression contains fractional order in various conclusions. As a result, choosing a good method to research resistor networks is important because it can simplify the problem and achieve simple results.

In this paper, an arbitrary $2 \times n$ plane resistor network, shown in figure 1, is investigated by the RT-I method, which can also be applied to a $\square \times n$ circuit network, shown in figure 2. As can be seen in the resistor network in figure 1, the same resistors r are located at the upper and lower horizontal lines, and the other resistors r_1 and r_0 are located at the middle horizontal line and the vertical lines respectively. The arbitrary resistance of this network with a different axis resistor has not been investigated before. The equivalent resistance between A_{x_1} and C_{x_2} and also the equivalent resistance between A_{x_1} and A_{x_2} are mainly studied here. As shown in figure 2, two asymmetric resistors R_1 and R are located at the horizontal axis grid, and the resistors R_0 are located at the vertical square's section. The resistors located at the horizontal line A_0A_n and line C_0C_n are R, and resistors located at the horizontal line B_0B_n and line D_0D_n are R_1. Here, we cleverly reduce the 3D network to the 2D network in order to study it.

2. Resistance of the 2 × *n* plane network

As shown in figure 1, the two asymmetric resistors located at the horizontal grid are set to be r_1 and r respectively, and resistors located at the vertical grid are set to be r_0. From left to right, we assume that A_k is the kth node at the axis A_0A_n and B_k is the kth node at the axis of B_0B_n. The resistance formulae between the two nodes of A_{x_1}, C_{x_2} and A_{x_1}, A_{x_2} in the arbitrary $2 \times n$ plane network are respectively

$$R_n(A_{x_1}, C_{x_2}) = br_1|x_2 - x_1| + \frac{r_0}{2}\left(\frac{\beta_{1,1}^{(1)} + 2\beta_{1,2}^{(1)} + \beta_{2,2}^{(1)}}{F_{n+1}^{(1)}} + b^2\frac{\beta_{1,1}^{(2)} - 2\beta_{1,2}^{(2)} + \beta_{2,2}^{(2)}}{F_{n+1}^{(2)}}\right). \tag{1}$$

$$R_n(A_{x_1}, A_{x_2}) = br_1|x_2 - x_1| + \frac{r_0}{2}\left(\frac{\beta_{1,1}^{(1)} - 2\beta_{1,2}^{(1)} + \beta_{2,2}^{(1)}}{F_{n+1}^{(1)}} + b^2\frac{\beta_{1,1}^{(2)} - 2\beta_{1,2}^{(2)} + \beta_{2,2}^{(2)}}{F_{n+1}^{(2)}}\right) \tag{2}$$

where $0 \leqslant x_1 \leqslant x_2 \leqslant n$, $b = r/(r + 2r_1)$, and the corresponding parameters are defined as

$$F_k^{(i)} = (\lambda_i^k - \bar{\lambda}_i^k)/(\lambda_i - \bar{\lambda}_i),\ (i = 1, 2)$$
$$\beta_{s,k}^{(i)} = \Delta F_{x_s}^{(i)}\Delta F_{n-x_k}^{(i)} \text{ with } \Delta F_k^{(i)} = F_{k+1}^{(i)} - F_k^{(i)}, \tag{3}$$

and

$$\lambda_i = \frac{1}{2}(t_i + \sqrt{t_i^2 - 4}),\quad \bar{\lambda}_i = \frac{1}{2}(t_i - \sqrt{t_i^2 - 4}) \tag{4}$$

with $h_1 = r_1/r_0$, $h = r/r_0$, and $t_i = 2 + h + h_1 + (-1)^i h_1$ which can be derived when $i = 1$ and 2

$$t_1 = 2 + h,\ t_2 = 2 + h + 2h_1. \tag{5}$$

Formulae (1) and (2) are interesting as they are two fractional-order resistances between two arbitrary nodes of a $2 \times n$ plane network.

In particular, when $x_1 = x_2 = x$ and $x_1 = 0$, $x_2 = n$, formulae (1) and (2) reduce to

$$R_n(A_x, C_x) = 2r_0\left(\frac{\Delta F_x^{(1)}\Delta F_{n-x}^{(1)}}{F_{n+1}^{(1)}}\right),\ (0 \leqslant x \leqslant n) \tag{6}$$

$$R_n(A_0, C_n) = \frac{rr_1n}{2r_1 + r} + r_0\left(1 - \frac{F_n^{(1)} - 1}{F_{n+1}^{(1)}}\right) + \frac{r_0r^2}{(2r_1 + r)^2}\left(1 - \frac{F_n^{(2)} + 1}{F_{n+1}^{(2)}}\right), \tag{7}$$

$$R_n(A_0, A_n) = \frac{rr_1n}{2r_1 + r} + r_0\left(1 - \frac{F_n^{(1)} + 1}{F_{n+1}^{(1)}}\right) + \frac{r_0r^2}{(2r_1 + r)^2}\left(1 - \frac{F_n^{(2)} + 1}{F_{n+1}^{(2)}}\right). \tag{8}$$

Please note that formulae (1), (2), and (6)–(8) are discovered for the first time in this paper. In the following section, we are going to derive the above formulae by the RT-I method

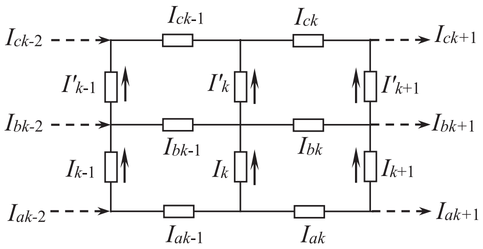

Figure 3. Segment of the network with different current directions and parameters.

[36–38] including to model differential equations and matrix transformation.

3. Derivation of the resistance formula

3.1. Building a general differential equation

A general differential equation model is built firstly to solve the $2 \times n$ plane network shown in figure 1. According to the structure characteristics of the network in figure 1, we assume a current I is injected into the network at point A_{x_1}, and exits at point B_{x_2} (or A_{x_2}). In order to establish the equation model, the $2 \times n$ plane network is shown in figure 3 with the current's parameters and directions represented. We assume the currents traversing the resistors of three horizontal lines are I_{ak}, I_{bk} and I_{ck} $(1 \leqslant k \leqslant n)$, respectively, and the resulting currents traversing resistors r_0 of all the $n+1$ vertical columns are I_k and I'_k $(0 \leqslant k \leqslant n)$, respectively.

Consider the segment of the $2 \times n$ plane network shown in figure 3. Using Kirchhoff's laws to analyze the resistor network, the node current equations and the mesh voltage equations can be achieved by the network in figure 3. Based on the four rectangular meshes and six nodes, the following equations, which only contain current parameters along the vertical direction, are obtained by eliminating the current parameters along the horizontal direction,

$$\begin{aligned} I_{k+1} &= (2+h+h_1)I_k - h_1 I'_k - I_{k-1}, \\ I'_{k+1} &= (2+h+h_1)I'_k - h_1 I_k - I'_{k-1}, \end{aligned} \tag{9}$$

where $h = r/r_0$, $h_1 = r_1/r_0$. Equation (9) can be written in a matrix form

$$\begin{pmatrix} I_{k+1} \\ I'_{k+1} \end{pmatrix} = \begin{pmatrix} 2+h+h_1 & -h_1 \\ -h_1 & 2+h+h_1 \end{pmatrix} \begin{pmatrix} I_k \\ I'_k \end{pmatrix} - \begin{pmatrix} I_{k-1} \\ I'_{k-1} \end{pmatrix}. \tag{10}$$

After the simple calculation, we can transform matrix (10) to be

$$\begin{pmatrix} X_{k+1}^{(1)} \\ X_{k+1}^{(2)} \end{pmatrix} = \begin{pmatrix} t_1 & 0 \\ 0 & t_2 \end{pmatrix} \begin{pmatrix} X_k^{(1)} \\ X_k^{(2)} \end{pmatrix} - \begin{pmatrix} X_{k-1}^{(1)} \\ X_{k-1}^{(2)} \end{pmatrix}, \tag{11}$$

where

$$t_1 = 2+h, \ t_2 = 2+h+2h_1, \tag{12}$$

and

$$\begin{pmatrix} X_k^{(1)} \\ X_k^{(2)} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} I_k \\ I'_k \end{pmatrix} = \begin{pmatrix} I_k + I'_k \\ I_k - I'_k \end{pmatrix}. \tag{13}$$

Making the inverse transform, it can be derived that

$$I_k = \frac{X_k^{(2)} + X_k^{(1)}}{2}, \quad I'_k = \frac{X_k^{(1)} - X_k^{(2)}}{2}. \tag{14}$$

The characteristic equation $x^2 = t_i x - 1 (i = 1, 2)$ can be known from (11); as a result, the characteristic roots of equation (4) with equation (5) can be obtained by assuming that λ_i, $\overline{\lambda}_i$ are the roots of the characteristic equation.

Solving equation (11), meanwhile considering the current condition at the input and output nodes, it can be known that

$$X_k^{(i)} = X_1^{(i)} F_k^{(i)} - X_0^{(i)} F_{k-1}^{(i)} \quad (0 \leqslant k \leqslant x_1) \tag{15}$$

$$X_k^{(i)} = X_{x_1+1}^{(i)} F_{k-x_1}^{(i)} - X_{x_1}^{(i)} F_{k-x_1-1}^{(i)} \quad (x_1 \leqslant k \leqslant x_2) \tag{16}$$

$$X_k^{(i)} = X_{x_2+1}^{(i)} F_{k-x_2}^{(i)} - X_{x_2}^{(i)} F_{k-x_2-1}^{(i)} \quad (x_2 \leqslant k \leqslant n) \tag{17}$$

where $F_k^{(i)}$ is defined in equation (3). Equations (15)–(17) are the general solution for any condition in the $2 \times n$ plane network, which will be used to solve the particular solution under different boundary conditions.

3.2. Constraint condition of the boundary

The constraint condition of the boundary current in the $2 \times n$ plane network has three parts, which contain the boundary condition constraint on the left and right, and the condition constraint of the current input and output node.

Considering the boundary conditions of the left edges in the resistor network, the resistor network can be analyzed by using Kirchhoff's current laws (KCL) and Kirchhoff's voltage law (KVL). Adopting a similar method to equation (10), the following matrix equation can be obtained to model the boundary conditions:

$$\begin{pmatrix} I_1 \\ I'_1 \end{pmatrix} = \begin{pmatrix} h+h_1+1 & -h_1 \\ -h_1 & h+h_1+1 \end{pmatrix} \begin{pmatrix} I_0 \\ I'_0 \end{pmatrix}, \tag{18}$$

where $h = r/r_0$, $h_1 = r_1/r_0$. Conducting the same matrix transform method as that in equations (10)–(11), we have

$$X_1^{(i)} = (t_i - 1) X_0^{(i)}, \ (i = 1, 2). \tag{19}$$

In addition, considering the boundary conditions of the right edges in the resistor network, the resistor network can be analyzed by using Kirchhoff's laws (KCL and KVL). Adopting a similar method to that used in establishing equation (10), the following formula can be obtained to model the boundary conditions:

$$\begin{pmatrix} I_n \\ I'_n \end{pmatrix} \begin{pmatrix} h+h_1+1 & -h_1 \\ -h_1 & h+h_1+1 \end{pmatrix} = \begin{pmatrix} I_{n-1} \\ I'_{n-1} \end{pmatrix}. \tag{20}$$

Conducting the matrix transform method as that in equation (11), we obtain

$$X_{n-1}^{(i)} = (t_i - 1) X_n^{(i)}, \ (i = 1, 2). \tag{21}$$

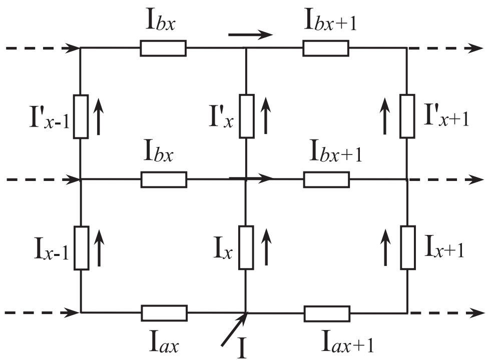

Figure 4. Segment of the network where the current injects the network at the node of A_{x1}.

Furthermore, considering the constraint equation of the current at the input node A_{x_1} as shown in figure 4, the following formula can be derived when the current enters the network from node A_{x_1} on the A_0A_n axis,

$$\begin{pmatrix} I_{x_1+1} \\ I'_{x_1+1} \end{pmatrix} = \begin{pmatrix} 2+h+h_1 & -h_1 \\ -h_1 & 2+h+h_1 \end{pmatrix}\begin{pmatrix} I_{x_1} \\ I'_{x_1} \end{pmatrix} - \begin{pmatrix} I_{x_1-1} \\ I'_{x_1-1} \end{pmatrix} - \begin{pmatrix} hI \\ 0 \end{pmatrix}, \tag{22}$$

where $h = r/r_0$, $h_1 = r_1/r_0$. Conducting the same matrix transform method as that in equation (11), from (22), we have

$$X^{(i)}_{x_1+1} = t_i X^{(i)}_{x_1} - X^{(i)}_{x_1-1} - hI. \tag{23}$$

The above equations are the general equations applicable to all situations and can be applied to calculate the equivalent resistance in various situations. According to different conditions of the output current, special constraint equations can be established respectively to derive equivalent resistance $R(A_{x_1},C_{x_2})$ and $R(A_{x_1},A_{x_2})$ in different situations.

3.3. Constraint equation under different output currents

Considering the constraint equation of the current at the output node, the following formula can be derived when the current exits the network from node C_{x_2} on the C_0 C_n axis,

$$\begin{pmatrix} I_{x_2+1} \\ I'_{x_2+1} \end{pmatrix} = \begin{pmatrix} 2+h+h_1 & -h_1 \\ -h_1 & 2+h+h_1 \end{pmatrix}\begin{pmatrix} I_{x_2} \\ I'_{x_2} \end{pmatrix} - \begin{pmatrix} I_{x_2-1} \\ I'_{x_2-1} \end{pmatrix} - \begin{pmatrix} 0 \\ hI \end{pmatrix}, \tag{24}$$

where $h = r/r_0$, $h_1 = r_1/r_0$. Conducting the same matrix transform method as that in equation (11), we have

$$X^{(i)}_{x_2+1} = t_i X^{(i)}_{x_2} - X^{(i)}_{x_2-1} + (-1)^i hI. \tag{25}$$

In addition, when the current exits the network from node A_{x_2} on the A_0A_n axis, it can be known

$$\begin{pmatrix} I_{x_2+1} \\ I'_{x_2+1} \end{pmatrix} = \begin{pmatrix} 2+h+h_1 & -h_1 \\ -h_1 & 2+h+h_1 \end{pmatrix}\begin{pmatrix} I_{x_2} \\ I'_{x_2} \end{pmatrix} - \begin{pmatrix} I_{x_2-1} \\ I'_{x_2-1} \end{pmatrix} + \begin{pmatrix} hI \\ 0 \end{pmatrix}. \tag{26}$$

Conducting the matrix transform to equation (26), it can be derived

$$X^{(i)}_{x_2+1} = t_i X^{(i)}_{x_2} - X^{(i)}_{x_2-1} + hI. \tag{27}$$

As a result, all essential equations for evaluating the resistance of the $2 \times n$ plane network have been obtained, and formulae (1) and (2) can be derived by the above equations.

3.4. Derivation of resistance $R_n(A_{x_1}, C_{x_2})$

Combining equations (15)–(17), (19), (21), (23) and (25), the solution of $X_k^{(i)}$ ($x_1 \leqslant k \leqslant x_2$) can be obtained after some algebra and reduction:

$$X_k^{(i)} = hI\frac{\Delta F^{(i)}_{x_1}\Delta F^{(i)}_{n-k} - (-1)^i \Delta F^{(i)}_k \Delta F^{(i)}_{n-x_2}}{(t_i - 2)F^{(i)}_{n+1}} \tag{28}$$

where t_i is given in equation (5). The voltage between the two nodes A_{x_1} and C_{x_2} is obtained via three different paths in figure 1, respectively

$$\begin{aligned} U(A_{x_1}C_{x_2}) &= \sum_{i=x_1+1}^{x_2} I_{ai}r + (I_{x_2} + I'_{x_2})r_0, \\ U(A_{x_1}C_{x_2}) &= \sum_{i=x_1+1}^{x_2} I_{bi}r_1 + (I_{x_1} + I'_{x_2})r_0, \\ U(A_{x_1}C_{x_2}) &= \sum_{i=x_1+1}^{x_2} I_{ci}r + (I_{x_1} + I'_{x_1})r_0. \end{aligned} \tag{29}$$

Deforming the above three equations and summarizing them in the same direction, it can be known

$$\left(\frac{2}{r} + \frac{1}{r_1}\right)U(A_{x_1}C_{x_2}) = \sum_{i=x_1+1}^{x_2}(I_{ai} + I_{bi} + I_{ci}) + \frac{r_0}{r}(I_{x_2} + I'_{x_2}) + \frac{r_0}{r_1}(I_{x_1} + I'_{x_2}) + \frac{r_0}{r}(I_{x_1} + I'_{x_1}).$$

According to the characteristic nature of the current, we have $I_{ai} + I_{bi} + I_{ci} = I$ when $x_1 \leqslant i \leqslant x_2$, and substituting to the above equation, we obtain

$$U(A_{x_1}C_{x_2}) = \frac{rr_1 I}{r + 2r_1}|x_2 - x_1| + \frac{r_0 r_1}{r + 2r_1} \times \left[I_{x_2} + \left(\frac{h_1 + h}{h_1}\right)I'_{x_2} + \left(\frac{h_1 + h}{h_1}\right)I_{x_1} + I'_{x_1}\right]. \tag{30}$$

Substituting equation (14) into equation (30), we have

$$U(A_{x_1}C_{x_2}) = \frac{rr_1 I}{r+2r_1}|x_2 - x_1| + \frac{r_0 r_0}{2(r+2r_1)} \times \begin{bmatrix} (2h_1+h)(X_{x_1}^{(1)} + X_{x_2}^{(1)}) \\ +h(X_{x_1}^{(2)} - X_{x_2}^{(2)}) \end{bmatrix}. \tag{31}$$

Taking $k = x_1, x_2$ in equation (28), it can be derived

$$X_{x_1}^{(i)} = hI\frac{\Delta F_{x_1}^{(i)}\Delta F_{n-x_1}^{(i)} - (-1)^i \Delta F_{x_1}^{(i)}\Delta F_{n-x_2}^{(i)}}{(t_i-2)F_{n+1}^{(i)}}, \tag{32}$$

$$X_{x_2}^{(i)} = hI\frac{\Delta F_{x_1}^{(i)}\Delta F_{n-x_2}^{(i)} - (-1)^i \Delta F_{x_2}^{(i)}\Delta F_{n-x_2}^{(i)}}{(t_i-2)F_{n+1}^{(i)}}, \tag{33}$$

where t_i is given in equation (5). Substituting equations (32) and (33) into equation (31), we obtain

$$\frac{U(A_{x_1}C_{x_2})}{I} = br_1|x_2 - x_1| + \frac{r_0}{2}\left(\frac{\beta_{1,1}^{(1)} + 2\beta_{1,2}^{(1)} + \beta_{2,2}^{(1)}}{F_{n+1}^{(1)}} + b^2\frac{\beta_{1,1}^{(2)} - 2\beta_{1,2}^{(2)} + \beta_{2,2}^{(2)}}{F_{n+1}^{(2)}}\right), \tag{34}$$

where $\beta_{k,s}^{(i)} = \Delta F_{x_k}^{(i)}\Delta F_{n-x_s}^{(i)}$ is defined in (3). It can be known that $R_n(A_{x_1}C_{x_2}) = U(A_{x_1}C_{x_2})/I$ by Ohm's law, and formula (1) is proved by (34) clearly.

3.5. Derivation of resistance $R_n(A_{x_1}, A_{x_2})$

Combining equations (15)–(17), (19), (21), (23) and (27), the solution of $X_k^{(i)}$ $(x_1 \leqslant k \leqslant x_2)$ can be obtained after some algebra and reduction

$$X_k^{(i)} = hI\frac{\Delta F_{x_1}^{(i)}\Delta F_{n-k}^{(i)} - \Delta F_k^{(i)}\Delta F_{n-x_2}^{(i)}}{(t_i-2)F_{n+1}^{(i)}}, \tag{35}$$

where t_i is given in equation (5). The voltage between two nodes A_{x_1} and A_{x_2} is obtained via three different paths in figure 3, respectively

$$\begin{aligned} U(A_{x_1}, A_{x_2}) &= \sum_{i=x_1+1}^{x_2} I_{ai} r, \\ U(A_{x_1}, A_{x_2}) &= \sum_{i=x_1+1}^{x_2} I_{bi} r_1 + I_{x_1} r_0 - I_{x_2} r_0, \\ U(A_{x_1}, A_{x_2}) &= \sum_{i=x_1+1}^{x_2} I_{ci} r + (I_{x_1} + I'_{x_1} - I'_{x_2} - I_{x_2}) r_0. \end{aligned} \tag{36}$$

Deforming the above three equations and summarizing them in the same direction, it can be known that

$$\left(\frac{2}{r} + \frac{1}{r_1}\right)U(A_{x_1}, A_{x_2}) = \sum_{i=x_1+1}^{x_2}(I_{ai} + I_{bi} + I_{ci}) + \frac{r_0}{r_1}(I_{x_1} - I_{x_2}) + \frac{r_0}{r}(I_{x_1} + I'_{x_1} - I'_{x_2} - I_{x_2})$$

.According to the characteristic nature of the current, it can be known that $I_{ai} + I_{bi} + I_{ci} = I$ when $x_1 \leqslant i \leqslant x_2$, and substituting to the above equation, it can be obtained

$$U(A_{x_1}, A_{x_2}) = \frac{rr_1 I}{r+2r_1}|x_2 - x_1| + \frac{r_0 r_0}{r+2r_1} \times [(h+h_1)(I_{x_1} - I_{x_2}) + h_1(I'_{x_1} - I'_{x_2})]. \tag{37}$$

Substituting equation (14) into equation (37), we obtain

$$U(A_{x_1}, A_{x_2}) = \frac{rr_1 I}{r+2r_1}|x_2 - x_1| + \frac{r_0}{2} \times \left[(X_{x_1}^{(1)} - X_{x_2}^{(1)}) + \frac{h}{h+2h_1}(X_{x_1}^{(2)} - X_{x_2}^{(2)})\right]. \tag{38}$$

Taking $k = x_1, x_2$ in (35), it can be derived that

$$X_{x_1}^{(i)} = hI\frac{\Delta F_{x_1}^{(i)}\Delta F_{n-x_1}^{(i)} - \Delta F_{x_1}^{(i)}\Delta F_{n-x_2}^{(i)}}{(t_i-2)F_{n+1}^{(i)}}, \tag{39}$$

$$X_{x_2}^{(i)} = hI\frac{\Delta F_{x_1}^{(i)}\Delta F_{n-x_2}^{(i)} - \Delta F_{x_2}^{(i)}\Delta F_{n-x_2}^{(i)}}{(t_i-2)F_{n+1}^{(i)}}, \tag{40}$$

where t_i is given in equation (5). Substituting equations (39) and (40) into equation (38), we obtain

$$\frac{U(A_{x_1}, A_{x_2})}{I} = br_1|x_2 - x_1| + \frac{r_0}{2} \times \left(\frac{\beta_{1,1}^{(1)} - 2\beta_{1,2}^{(1)} + \beta_{2,2}^{(1)}}{F_{n+1}^{(1)}} + b^2\frac{\beta_{1,1}^{(2)} - 2\beta_{1,2}^{(2)} + \beta_{2,2}^{(2)}}{F_{n+1}^{(2)}}\right), \tag{41}$$

where $\beta_{k,s}^{(i)} = \Delta F_{x_k}^{(i)}\Delta F_{n-x_s}^{(i)}$. It can be known that $R_n(A_{x_1}, A_{x_2}) = U(A_{x_1}, A_{x_2})/I$ by Ohm's law, and formula (1) is proved by (41) clearly.

4. Resistance of the □ × *n* cube network

Considering a $\square \times n$ circuit network as shown in figure 2, we set two asymmetric resistors of R_1 and R on the horizontal axis grid; the resistors on the vertical section square are R_0, where all the resistor elements on the horizontal axis of A_0A_n and C_0C_n are R, and all the resistor elements on the horizontal axis of B_0B_n and D_0D_n are R_1. We find that the structure of the resistor network has perfect anti-symmetry and rotational symmetry in figure 2 for nodes A_0A_n and C_0C_n when analyzing the structural characteristics of the resistor network model shown in figure 2. When current I goes from A_{x_1} to A_{x_2} (or to C_{x_2}), we have the same voltage for the symmetric nodes on the B_0B_n and D_0D_n axis, namely $U(B_k) = U(D_k)$. Thus, when we just calculate the equivalent resistance between two nodes of A_{x_1} and A_{x_2} or A_{x_1} and C_{x_2}, the 3D $\square \times n$ circuit network in figure 2 can be equivalent to the structure of the $2 \times n$ plane network model shown in figure 1.

Thus, formulae (1) and (2) can be applied to the 3D $\square \times n$ circuit network in figure 2 if we change the corresponding resistor elements in the different networks, such as

$$r_0 = R_0/2, \quad r_1 = R_1/2, \quad r = R. \tag{42}$$

According to the transformation between figures 1 and 2, and using (42), we have the relations

$$d_1 = R_1/R_0 = h_1, \quad d = R/R_0 = \frac{1}{2}h. \tag{43}$$

Thus, substituting equations (42) and (43) into equations (6)–(8), we have the equivalent resistance between two nodes of A_{x_1}, C_{x_2} and A_{x_1}, A_{x_2} in the arbitrary $\square \times n$ resistor network

$$R_{A_{x_1}C_{x_2}}(n) = \frac{aR_1}{2}|x_2 - x_1| + \frac{R_0}{4} \times \left(\frac{\beta_{1,1}^{(1)} + 2\beta_{1,2}^{(1)} + \beta_{2,2}^{(1)}}{F_{n+1}^{(1)}} + a^2\frac{\beta_{1,1}^{(2)} - 2\beta_{1,2}^{(2)} + \beta_{2,2}^{(2)}}{F_{n+1}^{(2)}}\right), \tag{44}$$

$$R_{A_{x_1}A_{x_2}}(n) = \frac{aR_1}{2}|x_2 - x_1| + \frac{R_0}{4} \times \left(\frac{\beta_{1,1}^{(1)} - 2\beta_{1,2}^{(1)} + \beta_{2,2}^{(1)}}{F_{n+1}^{(1)}} + a^2\frac{\beta_{1,1}^{(2)} - 2\beta_{1,2}^{(2)} + \beta_{2,2}^{(2)}}{F_{n+1}^{(2)}}\right), \tag{45}$$

where $0 \leqslant x_1 \leqslant x_2 \leqslant n$, $a = R/(R + R_1)$, and we redefine

$$F_k^{(i)} = (\mu_i^k - \overline{\mu}_i^k)/(\mu_i - \overline{\mu}_i), \quad (i = 1, 2)$$
$$\beta_{s,k}^{(i)} = \Delta F_{x_s}^{(i)} \Delta F_{n-x_k}^{(i)} \text{ with } \Delta F_k^{(i)} = F_{k+1}^{(i)} - F_k^{(i)}, \tag{46}$$

and

$$\mu_i = \frac{1}{2}(s_i + \sqrt{s_i^2 - 4}), \quad \overline{\mu}_i = \frac{1}{2}(s_i - \sqrt{s_i^2 - 4}) \tag{47}$$

with $d = R/R_0$, $d_1 = R_1/R_0$, and $s_i = 2 + 2d + d_1 + (-1)^i d_1$, such as

$$s_1 = 2 + 2d, \quad s_2 = 2 + 2d + 2d_1. \tag{48}$$

In particular, when $R_1 = R$, $x_1 = x_2 = x$, and $x_1 = 0$, $x_2 = n$, formulae (44) and (45) reduce to

$$\frac{R_n(A_x, C_x)}{R} = \frac{R_0}{R}\left(\frac{\Delta F_x^{(1)} \Delta F_{n-x}^{(1)}}{F_{n+1}^{(1)}}\right), \tag{49}$$

$$\frac{R_{A_0A_n}(n)}{R} = \frac{n}{4} + \frac{R_0}{2R}\left(1 - \frac{F_n^{(1)} + 1}{F_{n+1}^{(1)}}\right) + \frac{R_0}{8R}\left(1 - \frac{F_n^{(2)} + 1}{F_{n+1}^{(2)}}\right), \tag{50}$$

$$\frac{R_{A_0C_n}(n)}{R} = \frac{n}{4} + \frac{R_0}{2R}\left(1 - \frac{F_n^{(1)} - 1}{F_{n+1}^{(1)}}\right) + \frac{R_0}{8R}\left(1 - \frac{F_n^{(2)} + 1}{F_{n+1}^{(2)}}\right), \tag{51}$$

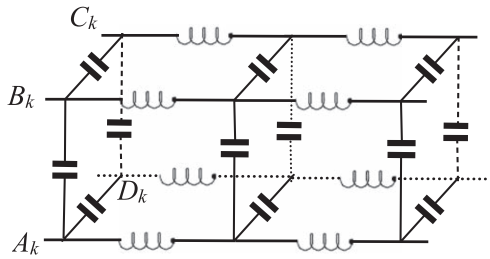

Figure 5. Segment of the *LC* network with parameters L and C.

where $F_k^{(i)}$ is defined in equation (46), and μ_i, $\overline{\mu}_i$ is defined in equation (47) with (48) and $d_1 = d = R/R_0$.

5. Equivalent impedance of the □ × *n* *LC* network

5.1. Impedance formula of the LC network

The grid elements in figure 2 are arbitrary which can be either a resistor or an impedance. We consider a $\square \times n$ complex impedance network, such as its segment grid of the impedance network shown in figure 5. Assuming the ac frequency is ω, then there must exist three mapping relations:

$$R_0 = \frac{1}{j\omega C}, \quad R = j\omega L, \quad d = \frac{R}{R_0} = -\omega^2 LC \tag{52}$$

where $j^2 = -1$. By substituting (52) into (49), (50) and (51), we obtain the equivalent impedance of a $\square \times n$ circuit network. Thus the general formula of the equivalent impedances between two nodes in a $\square \times n$ *LC* network can be written as

$$\frac{R_n(A_x, C_x)}{j\omega L} = \frac{-1}{\omega^2 LC}\left(\frac{\Delta F_x^{(1)} \Delta F_{n-x}^{(1)}}{F_{n+1}^{(1)}}\right), \tag{53}$$

$$\frac{Z_{A_0A_n}(n)}{j\omega L} = \frac{1}{4}n - \frac{1}{2\omega^2 LC}\left[\left(1 - \frac{F_n^{(1)} + 1}{F_{n+1}^{(1)}}\right) + \frac{1}{4}\left(1 - \frac{F_n^{(2)} + 1}{F_{n+1}^{(2)}}\right)\right] \tag{54}$$

$$\frac{Z_{A_0C_n}(n)}{j\omega L} = \frac{1}{4}n - \frac{1}{2\omega^2 LC}\left[\left(1 - \frac{F_n^{(1)} - 1}{F_{n+1}^{(1)}}\right) + \frac{1}{4}\left(1 - \frac{F_n^{(2)} + 1}{F_{n+1}^{(2)}}\right)\right] \tag{55}$$

where $F_k^{(i)} = (\mu_i^k - \overline{\mu}_i^k)/(\mu_i - \overline{\mu}_i)$ is defined in equation (46). By (47), (48) and (52) we have

$$\mu_1 = 1 - \omega^2 LC + \omega\sqrt{LC(\omega^2 LC - 2)},$$
$$\overline{\mu}_1 = 1 - \omega^2 LC - \omega\sqrt{LC(\omega^2 LC - 2)} \tag{56}$$

and

$$\mu_2 = 1 - 2\omega^2 LC + 2\omega\sqrt{LC(\omega^2 LC - 1)},$$
$$\overline{\mu}_2 = 1 - 2\omega^2 LC - 2\omega\sqrt{LC(\omega^2 LC - 1)}. \tag{57}$$

Obviously the characteristics of the equivalent impedances of $Z(n)$ will be determined by the corresponding function of $F_n^{(i)}$ associated with equations (56) and (57).

Formulae (53)–(55) may be plural since there may be $\omega < 1/\sqrt{LC}$, which is therefore more valuable and can be applied more than the resistance formula. By investigation, we find that equations (53)–(55) are always real numbers when $\omega > \sqrt{2}/\sqrt{LC}$. But when $\omega \leqslant 1/\sqrt{LC}$, they are always plural. So it is not enough to just give equations (53)–(55); we have to discuss them in terms of different conditions.

5.2. Oscillation and resonance of the LC network

Case 1. $\omega = \sqrt{2/LC}$

When $\omega = \sqrt{2/LC}$, from (56), we have $\mu_1 = \bar{\mu}_1 = -1$, and taking the limit to $F_n^{(1)}$, yields

$$\lim_{\mu_1 \to \bar{\mu}_1 = -1} F_n^{(1)} = \frac{\mu_1^n - \bar{\mu}_1^n}{\mu_1 - \bar{\mu}_1} = n(-1)^{n-1}. \tag{58}$$

Substituting equation (58) into equations (53)–(55), we have

$$\frac{Z_{A_xC_x}(n)}{j\omega L} = -\frac{1}{\omega^2 LC}\left(\frac{(2x+1)(2n-2x+1)}{n+1}\right), \tag{59}$$

$$\frac{Z_{A_0A_n}(n)}{j\omega L} = \frac{1}{4}n - \frac{1}{4}\left[2 - \frac{1+(-1)^n}{n+1} + \frac{1}{4}\left(1 - \frac{F_n^{(2)}+1}{F_{n+1}^{(2)}}\right)\right] \tag{60}$$

$$\frac{Z_{A_0C_n}(n)}{j\omega L} = \frac{1}{4}n - \frac{1}{4}\left[2 - \frac{1-(-1)^n}{n+1} + \frac{1}{4}\left(1 - \frac{F_n^{(2)}+1}{F_{n+1}^{(2)}}\right)\right] \tag{61}$$

where $F_k^{(2)} = (\mu_2^k - \bar{\mu}_2^k)/(\mu_2 - \bar{\mu}_2)$, and

$$\mu_2 = -3 + 2\sqrt{2},\ \bar{\mu}_2 = -3 - 2\sqrt{2}. \tag{62}$$

Case 2. $1/\sqrt{LC} < \omega < \sqrt{2}/\sqrt{LC}$

When $1/\sqrt{LC} < \omega < \sqrt{2}/\sqrt{LC}$, from (56), we have

$$\begin{cases} \mu_1 = \cos\theta + j\sin\theta \\ \bar{\mu}_1 = \cos\theta - j\sin\theta \end{cases} \tag{63}$$

where $\theta = \arccos(1 - \omega^2 LC)$, $j^2 = -1$. So we have

$$F_n^{(1)} = \frac{\mu_1^n - \bar{\mu}_1^n}{\mu_1 - \bar{\mu}_1} = \frac{\sin n\theta}{\sin\theta}. \tag{64}$$

Substituting equation (64) into equations (53)–(55), we have

$$\frac{Z_{A_xC_x}(n)}{j\omega L} = -\frac{\cos(x+1)\theta + \cos(n-x)\theta}{\sin(n+1)\theta\sin\theta}, \tag{65}$$

$$\frac{Z_{A_0A_n}(n)}{j\omega L} = \frac{1}{4}n - \frac{1}{2\omega^2 LC}\left[\left(1 - \frac{\sin n\theta + \sin\theta}{\sin(n+1)\theta}\right) + \frac{1}{4}\left(1 - \frac{F_n^{(2)}+1}{F_{n+1}^{(2)}}\right)\right], \tag{66}$$

$$\frac{Z_{A_0C_n}(n)}{j\omega L} = \frac{1}{4}n - \frac{1}{2\omega^2 LC}\left[\left(1 - \frac{\sin n\theta - \sin\theta}{\sin(n+1)\theta}\right) + \frac{1}{4}\left(1 - \frac{F_n^{(2)}+1}{F_{n+1}^{(2)}}\right)\right], \tag{67}$$

where $\theta = \arccos(1 - \omega^2 LC)$, and $F_n^{(2)}$ is the real number. Equations (65)–(67) show that the equivalent impedance has oscillation characteristics with the change of n or $\omega^2 LC$.

Case 3. $\omega < 1/\sqrt{LC}$

When $\omega < 1/\sqrt{LC}$, we have (63) and (64), and also

$$\begin{cases} \mu_2 = \cos\varphi + j\sin\varphi \\ \bar{\mu}_2 = \cos\varphi - j\sin\varphi \end{cases} \tag{68}$$

where $\varphi = \arccos(1 - 2\omega^2 LC)$. By $F_k^{(i)} = (\mu_i^k - \bar{\mu}_i^k)/(\mu_i - \bar{\mu}_i)$ we have

$$F_n^{(1)} = \frac{\sin n\theta}{\sin\theta},\quad F_n^{(2)} = \frac{\sin n\varphi}{\sin\varphi}. \tag{69}$$

Substituting equation (69) into equations (53)–(55), we have

$$\frac{Z_{A_xC_x}(n)}{j\omega L} = -\frac{\cos(x+1)\theta + \cos(n-x)\theta}{\sin(n+1)\theta\sin\theta}, \tag{70}$$

$$\frac{Z_{A_0A_n}(n)}{j\omega L} = \frac{1}{4}n - \frac{1}{2\omega^2 LC}\left[\left(1 - \frac{\sin n\theta + \sin\theta}{\sin(n+1)\theta}\right) + \frac{1}{4}\left(1 - \frac{\sin n\varphi + \sin\varphi}{\sin(n+1)\varphi}\right)\right] \tag{71}$$

$$\frac{Z_{A_0C_n}(n)}{j\omega L} = \frac{1}{4}n - \frac{1}{2\omega^2 LC}\left[\left(1 - \frac{\sin n\theta - \sin\theta}{\sin(n+1)\theta}\right) + \frac{1}{4}\left(1 - \frac{\sin n\varphi + \sin\varphi}{\sin(n+1)\varphi}\right)\right] \tag{72}$$

where $\theta = \arccos(1 - \omega^2 LC)$, $\varphi = \arccos(1 - 2\omega^2 LC)$.

From equations (70)–(72), we can guess that the three equivalent complex impedances have oscillation characteristics, but we cannot clearly understand the oscillation position. To understand the complex impedance characteristics intuitively, we plot equations (70)–(72) as shown in figures 6–8, where we denote $Z(D)/(j\omega L) = Z(D)/R$.

From the three 3D graphs of figures 6–8, it can be clearly seen that the three equivalent complex impedances have oscillation characteristics in a particular location.

Obviously, when we study the characteristics of equations (70)–(72), we find that the characteristic of $Z(n)$ changes unsteadily with the change of n and ω. The above equations are the personalized formulae under different conditions, which demonstrate that the analysis formula of the equivalent impedance in the complex impedance network is

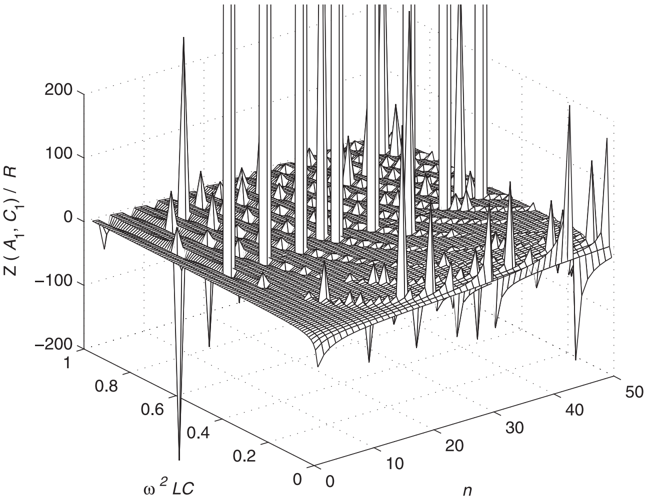

Figure 6. A 3D graph showing that the impedance of Z(A_1, C_1) changes with $\omega^2 LC$ and n in the case of $\omega^2 LC < 1$. Several rays (pillar, or branching pole) indicate their resonances under certain conditions of $\omega^2 LC = 0.5$. The bamboo shoots affirm that they are oscillating in that position.

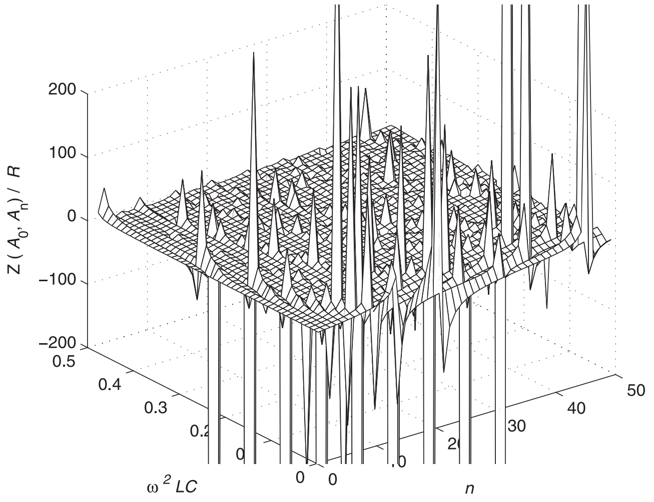

Figure 7. A 3D graph showing that the impedance of Z(A_0, A_n) changes with $\omega^2 LC$ and n in the case of $\omega^2 LC < 1/2$. Several rays (pillar, or branching pole) indicate their resonances in that position. The bamboo shoots affirm that they are oscillating in that position.

complicated, which is different from the equivalent resistance in the resistor network.

6. Summary and discussion

This study first investigated an arbitrary $2 \times n$ plane resistor network by the RT-I method, and obtained two general formulae of equations (1) and (2) in terms of fraction expression, which is found for the first time. In particular, we provide three special results when taking $x_1 = x_1 = x$ and $x_1 = 0$, $x_2 = n$. It is interesting that we established a new method to study the equivalent resistance of a 3D network, which is a very clever design that applies the results of a $2 \times n$ resistor network to a $\square \times n$ circuit network. It is necessary for us to explain that figure 2 is a new model because there are two different resistor elements on the horizontal axis, which has not been resolved before, and the general results of equations (44) and (45) are discovered for the first time.

As an application, the resistance conclusions obtained in this article are also applicable to the complex impedance network. We therefore consider the $\square \times n$ LC network shown in figure 5, and a series of equivalent impedance results are given under different conditions, such as three general results of equations (53)–(55), the three particular results of equations (59)–(61) in the case of $\omega = \sqrt{2/LC}$, the three particular results of equations (65)–(67) in the case of $\sqrt{1/LC} < \omega < \sqrt{2/LC}$, and the three particular results of equations (70)–(72) in the case of $\omega < \sqrt{1/LC}$. From equations (70)–(72), we cannot clearly understand the oscillation position of the three equivalent complex impedances, and we therefore plot equations (70)–(72) as shown in figures 6–8. Obviously, when we study the characteristics of (70)–(72), we find the characteristic of $Z(n)$ changes unsteadily with the change of n and ω, which shows that the analysis formula of the equivalent impedance in the complex impedance network is complicated, which is different from the equivalent resistance in the resistor network. This somewhat curious result suggests the possibility of the practical application of our formulae to resonant circuits. For example, a new method is established which can reduce a 3D cube network to a plain network. Several new theoretical results can be used for mathematical modeling research and the simulation of physical problems. It is helpful to study the structure and electrical properties of crystals, and provide a theoretical basis for microwave research.

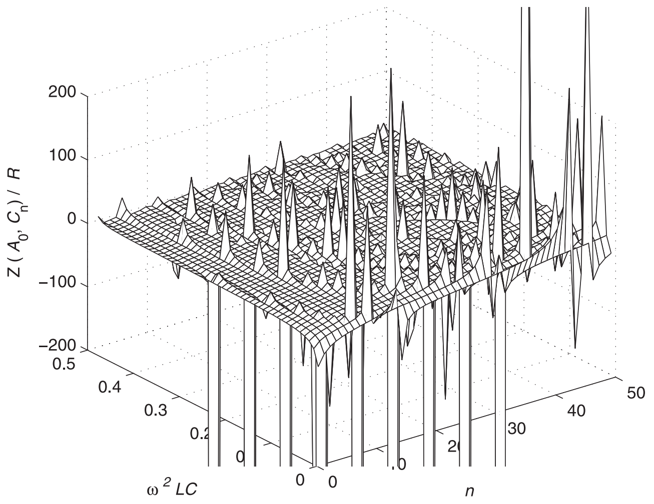

Figure 8. A 3D graph showing the impedance of Z(A_0, C_n) changes with $\omega^2 LC$ and n in the case of $\omega^2 LC < 1/2$. Several rays (pillar, or branching pole) indicate their resonances in that position. The bamboo shoots affirm that they are oscillating in that position.

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